Projects for Differential Algebra
Due Wednesday, December 26, 2018, 11:59 pm EST

Choose one question from the list below, prepare your answer in LaTeX or write it very clearly by hand, produce a PDF of your answer, and send it by e-mail to aovchinnikov@qc.cuny.edu by the due date above. Do not forget to provide all details in the proofs. "One can show...", "a calculation shows...", "one may assume..." must be avoided.

1. Find the websites of James Freitag, Omar León Sánchez, Alexey Ovchinnikov, Gleb Pogudin, Michael Singer, and Michael Wibmer, download the papers from there dated in the last three years, carefully read their introductions, and, based on this, present a list of important open problems related to differential algebra and its applications, explain the importance of the problems, and describe the partial cases in which these problems have been solved.

2. Prove that $\langle y'^2 - y\rangle^{(\infty)} : y'^\infty \not\subset \langle y\rangle^{(\infty)}$ but $\langle y'^2 - y^3\rangle^{(\infty)} : y'^\infty \subset \langle y\rangle^{(\infty)}$. Based on this, prove or disprove that

   (a) $\sqrt{\langle y'^2 - y\rangle^{(\infty)}}$ is prime,

   (b) $\sqrt{\langle y'^2 - y^3\rangle^{(\infty)}}$ is prime.

   Hint: among the papers by Evelyne Hubert, find the right one to use. All algorithms and statements from Hubert's paper that are used must be clearly quoted. It is also recommended to watch this video (talk by Michael Singer at the Kolchin Seminar on November 7, 2014):
   `https://www.youtube.com/watch?v=DxnUPL9Bsl0`

   `https://www.youtube.com/watch?v=ifWECx1gK-o`

3. Let $I \subset K[y_1,\ldots,y_n]$ be a prime ideal. Prove that $\sqrt{\langle I\rangle^{(\infty)}} \subset K[y_1,\ldots,y_n]_\infty$ is prime.

   Hint: it would be a good idea to review an article by Evelyne Hubert dated 2000, which is available for download at her website, and understand what statements from there give you all polynomials in the original input system that have order zero.

4. Describe and justify an algorithm (based on the Rosenfeld-Gröbner algorithm), with input $F \subset K[y_1,\ldots,y_n]_\infty$ such that the ideal $I := \sqrt{\langle F\rangle^\infty}$ is prime, that finds a characteristic set C of I (so that $I = \langle C\rangle^\infty : H_C^\infty$).

5. Download the paper: R. May and W. Leonard, "Nonlinear Aspects of Competition Between Three Species", SIAM Journal on Applied Mathematics, Vol. 29, No. 2 (1975), pp. 243-253, using, for example, the GC library credentials, present the 3-species ODE model from page 245, and determine using the existing software (search on the web for this) whether each of the parameters in the model is globally identifiable or locally identifiably or non-identifiable.

6. Prepare a 30-minute presentation on any part of the following papers (to present on December 14, from 11:45 am–1:45 pm):

(a) https://doi.org/10.1017/S1751731117002774

(b) https://doi.org/10.1371/journal.pone.0027755

(c) https://arxiv.org/abs/1712.01412

(d) https://arxiv.org/abs/1801.08112

(e) https://arxiv.org/abs/1610.04022

(f) https://arxiv.org/abs/1610.04353

(g) https://arxiv.org/abs/1602.00246

(h) https://arxiv.org/abs/1606.08492

7. Produce lecture notes for the course.