Queens College
Department of Mathematics

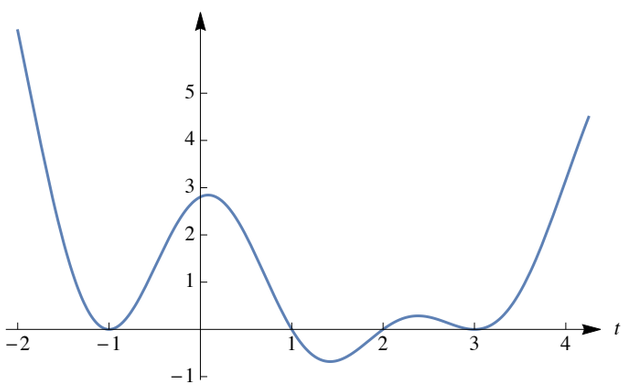
Math 142 | Final Examination $2\frac{1}{2}$ hours | Fall 2015

Part I: Short Answer

Define a function A by

$$A(x) = \int_0^x f(t)\,dt \text{ for } -2 \le x \le 4$$

where f is the function whose graph is sketched below:



Use the picture to answer questions 1–7.

1. What is $A(0)$?

2. What is $A'(0)$?

3. On which intervals is A increasing?

4. At which points does A have a local minimum?

5. Is $A(2)$ positive, negative, or zero?

6. Is $A(-1)$ positive, negative, or zero?

7. Is A concave up, concave down, or neither at 2?

Part II: Matching

Match the expression on the left with the appropriate choice on the right. Show your work.

8. $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$

9. $\displaystyle\int_0^1 \frac{1}{1+x}\,dx$

10. $\displaystyle\lim_{x\to 1^-} \int_0^x \frac{1}{\sqrt{1-t^2}}\,dt$

11. $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx$

12. $\displaystyle\lim_{x\to 1^-} \int_0^x \frac{t}{\sqrt{1-t^2}}\,dt$

13. $\displaystyle\int_{\sqrt{e}}^{e^2} \frac{1}{x\ln(x)}\,dx$

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{2}$

(c) $\frac{\ln(2)}{2}$

(d) $\ln(2)$

(e) $2\ln(2)$

(f) 1

(Continued on the Other Side)

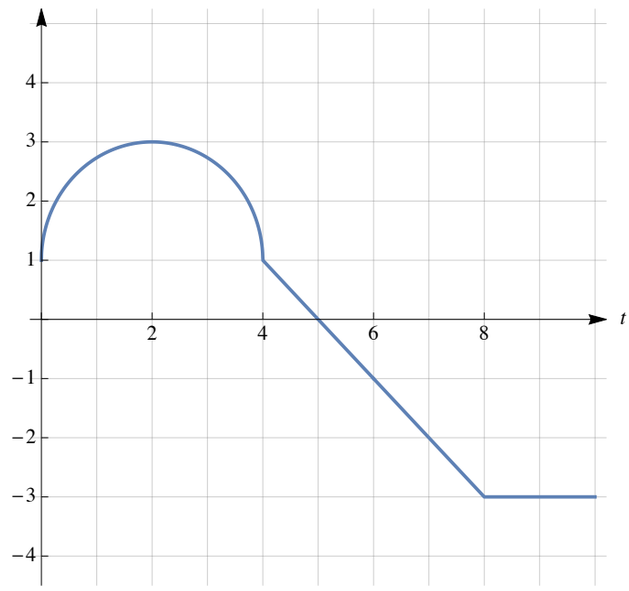
Part III: Multiple Choice

Show your work.

14. Approximating $\int_0^4 (x^3 - 6x^2 + 11x - 6)\,dx$ with a Riemann sum using $n = 4$ equal subdivisions and righthand endpoints yields:

(a) 6 (b) $\frac{8}{3}$ (c) 0 (d) $-\frac{8}{3}$ (e) -38

15. Which equals $\int_2^8 f(t)dt$ where f is the function sketched below ?



(a) $\pi + 6$ (b) $\pi + 4$ (c) $\pi + 2$ (d) π (e) $\pi - 2$

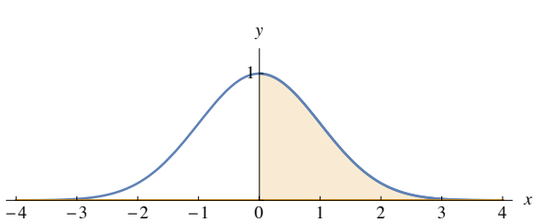
16. Let $r(x) = x^3 + 2x + 3$. Since $r'(x) = 2 + 3x^2 > 0$, the function r is strictly increasing, hence invertible. If s denotes the inverse of r, then $s'(3) =$

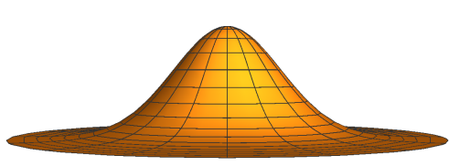
(a) -3 (b) -2 (c) 0 (d) $\frac{1}{2}$ (e) 3

17. Which are solutions to the differential equation $y'' + 4y' + 4y = 0$? There is more than one answer—indicate all that apply.

(a) $y = e^{-2x}$ (b) $y = 6e^{2x}$ (c) $y = 3xe^{x}$ (d) $y = 5xe^{-2x}$ (e) $y = xe^{2x}$

18. Consider the region under the bell curve $y = e^{-x^2/2}$ and above the x-axis between $x = 0$ and $x = 4$ and the solid obtained by rotating this region around the y-axis. Find the volume of this solid.





(a) $4\pi - \frac{4\pi}{\sqrt{e}}$ (b) $2e - \frac{2e}{\sqrt{\pi}}$ (c) $\frac{e^2}{\sqrt{2\pi}}$ (d) $2\pi - \frac{2\pi}{e^8}$ (e) $e - \frac{\sqrt{\pi}}{e^2}$