Answers

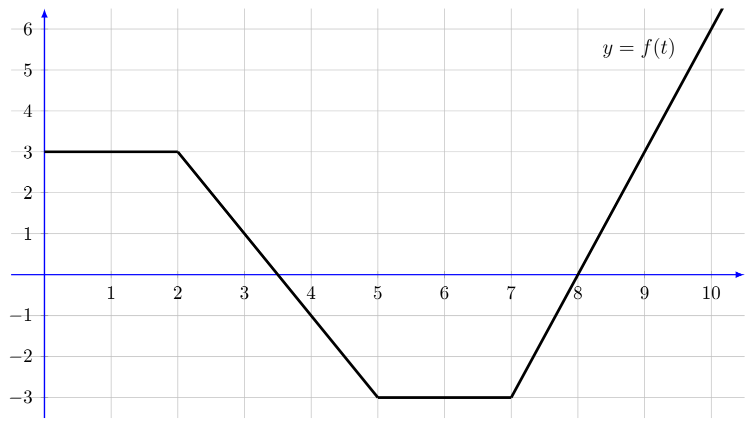
1	-6
2	0
3	2
4	$(0, 3.5)$ and $(8, 10)$
5	$x = 8$
6	$(7, 10)$

7	a
8	b
9	c
10	d
11	e
12	F
13	T
14	T
15	F
16	T

Part I: Use the picture

The graph of a function f is sketched below. Define an "Area function" A by

$$A(x) = \int_2^x f(t)dt$$



Use the picture and the definition of A to answer these questions:

1. What is $A(0)$?

Answer. $A(0) = \int_2^0 f(t)dt = -6$

2. What is $A(2)$?

Answer. $A(2) = \int_2^2 f(t)\,dt = 0$

3. What is $A(4)$?

Answer. $A(4) = \int_2^4 f(t)\,dt = 2$

4. Where is A increasing?

Answer. A is increasing where $A'(x) = f(x) > 0$ which is the interval $(0, 3.5)$ and $(8, 10)$.

5. Where is the minimum of A?

Answer. A has one critical point at $x = 8$ which is a global minimum.

6. Where is A concave up?

Answer. A is concave up where $A'(x) = g(x)$ is increasing, which is on the interval $(7, 10)$ (the interval $(5, 10)$ is also acceptable.)

Part II: Matching

Match the expression on the left with the appropriate choice on the right. Put the letter of the choice on the answer sheet.

7. $\displaystyle\int_0^8 |2x-6|\,dx = 34$

8. $\displaystyle\int_0^2 2x^2 - 3\,dx = -\frac{2}{3}$

9. $\displaystyle\int_0^1 x + \sqrt{1-x^2}\,dx = \frac{\pi+2}{4}$

10. $\displaystyle\int_0^{\frac{\sqrt{\pi}}{2}} 2x\sin(x^2)\,dx = 1 - \frac{1}{\sqrt{2}}$

11. $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2\left(\frac{1}{n}\right) = \frac{1}{3}$

(a) 34

(b) $-\dfrac{2}{3}$

(c) $\dfrac{\pi+2}{4}$

(d) $1-\dfrac{1}{\sqrt{2}}$

(e) $\dfrac{1}{3}$

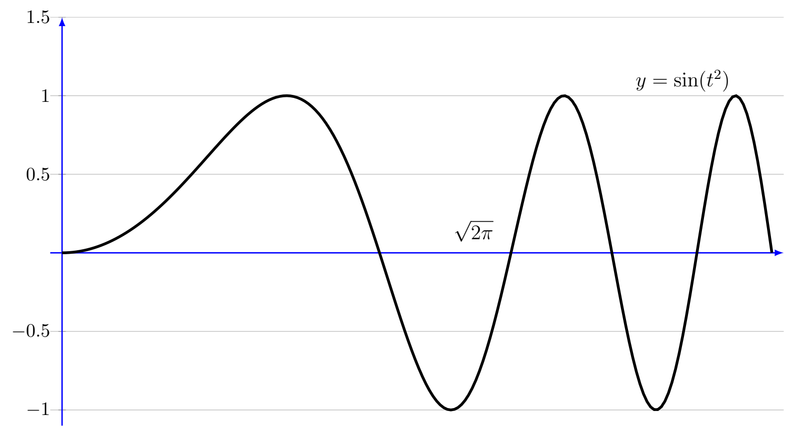
Hints:

- For 7, either sketch the graph and think in terms of area, or, use $|2x-6| = \begin{cases} 2x-6 & x>3 \\ -2x+6 & x<3 \end{cases}$ and break the interval $[0,8]$ into $[0,3]\cup[3,8]$ and
- For 8, find an antiderivative and use FTC.
- For 9, break into a sum and think of area since the curve $y = \sqrt{1-x^2}$ is part of a circle.
- For 10, use u-substitution.
- For 11, either recognize the sum as $\int_0^1 x^2$ or compute directly using $\sum_{i=1}^n i^2 = \frac{(n)(n-1)(2n+1)}{6}$.

Part III: True or False

12. The function g defined by $g(x) = \int_0^x \sin(t^2)\,dt$ has an absolute maximum at $x = \sqrt{2\pi}$.

Answer. False. g has critical points where $g'(x) = \sin(x^2) = 0$, namely $x = 0, \pm\sqrt{\pi}, \pm\sqrt{2\pi}, \ldots$



You can see $\sqrt{2\pi}$ is a local min since $g'(x) < 0$ to the left of $\sqrt{\pi}$ and $g'(x) > 0$ to the right. This problem is closely related to example 4 on page 236 of the text.

13. $\dfrac{d}{dx}\left(\int_1^3 \sin(t^2)\,dt\right) = 0.$

Answer. True. $\int_1^3 \sin(t^2)\,dt$ is a number, hence it's derivative with respect to x is zero.

14. If f' is continuous, then $\int_1^3 f'(v)\,dv = f(3) - f(1)$.

Answer. True. This follows from the evaluation theorem part of the FTC.

15. If f and g are integrable on the interval $[a, b]$ then

$$\int_a^b f(x)g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx.\right).$$

Answer. False. For example,

$$\int_0^1 (x)(x)\,dx = \frac{1}{3} \text{ and } \left(\int_0^1 x\,dx\right)\left(\int_0^1 x\,dx.\right) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}.$$

16. If f and g are integrable on the interval $[a, b]$ then

$$\int_a^b [5f(x) + g(x)]\,dx = 5\int_a^b f(x)\,dx + \int_a^b g(x)\,dx.$$

Answer. True. This follows from the linearity of the integral.