Name ____

1. Define $\ln(x)$.

Answer. For any $x > 0$, $\ln(x) := \int_1^x \frac{1}{t}\,dt$.

2. Define the number e.

Answer. e is the unique number satisfying $\ln(e) = 1$.

3	b
4	c
5	a
6	c
7	a
8	a
9	d
10	d

3. The curve $y = e^{-2x^2+10x}$ has two inflection points. One is at $x = 3$ and the other is at $x =$

(a) 0 (b) 2 (c) 2.5 (d) $\sqrt{5}$ (e) 5

Answer. (b). $y' = (-4x+10)e^{-2x^2+10x}$ and $y'' = (-4x)e^{-2x^2+10x} + (-4x+10)^2e^{-2x^2+10x}$ which simplifies to $y'' = 16(x-2)(x-3)e^{-2x^2+10x}$. The inflection points are $x = 2$ and $x = 3$.

4. The domain of $g(x) = \dfrac{e^x}{\ln(x+1)-1}$ is

(a) all real numbers $x > 0$

(b) all real numbers x except $e - 1$

(c) all real numbers $x > -1$ except $e - 1$

(d) $-1 < x < e - 1$

(e) all real numbers x except $x = -1$

Answer. (c). The domain consists of all real numbers x except for those that would have you taking the log of a negative, or dividing by zero. So, $x + 1 > 0 \Leftrightarrow x > -1$ and $\ln(x+1) - 1 \neq 0 \Leftrightarrow x + 1 \neq e$.

5. $\displaystyle\int_0^2 xe^{x^2}\,dx =$

(a) $\dfrac{e^4-1}{2}$ (b) 2 (c) $e^2 - 1$ (d) $\dfrac{1}{2}e^4$

Answer. (a). A u-substitution of $u = x^2$ will give you $\displaystyle\int_0^2 xe^{x^2}\,dx = \frac{1}{2}e^{x^2}\Big|_0^2 = \frac{e^4-1}{2}$.

6. The function f defined by $f(x) = \displaystyle\int_1^x \sqrt{3+t^4}\,dt$ is invertible and $(f^{-1})'(0) =$

(a) $-\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) $\dfrac{1}{2}$ (d) $\sqrt{3}$ (e) 2

Answer. (c). Note $f(1) = 0$, so $\left(f^{-1}\right)'(0) = \frac{1}{f'(1)} = \frac{1}{\sqrt{1+1^4}} = \frac{1}{2}$.

7. $\displaystyle\lim_{x\to 0^-} \exp\left(\frac{1}{x}\right) =$

(a) 0 (b) 1 (c) e (d) ∞ (e) $-\infty$

Answer. (a). As $x \to 0^-$, $\frac{1}{x} \to -\infty$ and $\exp(x) \to 0$ as $x \to -\infty$

8. $\displaystyle\int \ln(x)\,dx =$

(a) $x\ln(x) - x + C$

(b) $\dfrac{1}{x} + C$

(c) $\dfrac{(\ln(x))^2}{2} + C$

(d) $e^x + C$

(e) $x\ln(x) + C$

Answer. (a). Which can be checked by differentiating $x\ln(x) - x$:

$$(1)(\ln(x)) + (x)\left(\frac{1}{x}\right) - 1 = \ln(x)$$

9. The absolute minimum value of $h(x) = x^2\ln(x)$ is

(a) $-\dfrac{1}{\sqrt{e}}$ (b) $\sqrt{e}$ (c) $\dfrac{2}{e}$ (d) $-\dfrac{1}{2e}$ (e) $-2e$

Answer. (d). Differentiating $h'(x) = 2x\ln(x) + x^2\frac{1}{x} = 2x\ln(x) + x = x(2\ln(x) + 1)$ and we see $h'(x) = 0$ when $2\ln(x) + 1 = 0 \Rightarrow \ln(x) = -\frac{1}{2} \Rightarrow x = e^{-1}2$. This critical point is a minimum since $h'(x)$ is negative for smaller values of x and $h'(x)$ is positive for greater values of x. Substituting the critical point into $h(x)$ yields the minimum value of $-\frac{1}{2e}$

10. $\int \tan(x)\,dx =$

(a) $\cot(x) + C$

(b) $\sec(x) + C$

(c) $\sec^2(x) + C$

(d) $-\ln(\cos(x)) + C$

Answer. (d). Write $\tan(x) = \frac{\cos(x)}{\sin(x)}$ and let $u = \sin(x)$...