Compute

1. $\int_0^1 \frac{1}{1+x} = \ln(1+x)\Big|_0^1 = \ln(2).$

2. $\int_0^1 \frac{1}{1+x^2}\,dx = \arctan(x)\Big|_0^1 = \frac{\pi}{4}$

3. $\int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\log(1+x^2)\Big|_0^1 = \frac{\ln(2)}{2}.$

hint: Use u substitution $u = 1 + x^2$

4. $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)\Big|_0^1 = \frac{\pi}{2}.$

5. $\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}\Big|_0^1 = 1\ .$

hint: Use u substitution $u = 1 - x^2$

6. $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arcsinh}(x)\Big|_0^1 = \operatorname{arcsinh}(1)\ .$

7. $\int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx = \sqrt{1+x^2}\Big|_0^1 = \sqrt{2} - 1.$

hint: Use u substitution $u = 1 + x^2$

Compute

8. $\sinh(\ln(2)) = \frac{1}{2}\left(e^{\ln 2} - e^{-\ln 2}\right) = \frac{1}{2}\left(2 - \frac{1}{2}\right) = \frac{3}{4}$.

9. $\cosh(\ln(2)) = \frac{1}{2}\left(e^{\ln 2} + e^{-\ln 2}\right) = \frac{1}{2}\left(2 + \frac{1}{2}\right) = \frac{5}{4}$.

10. $\arcsin(1) = \frac{\pi}{2}$.

11. $\arctan(1) = \frac{\pi}{4}$.

12. $\operatorname{arcsinh}\left(\frac{3}{4}\right) =$ the unique number whose sinh is $\frac{3}{4} = \ln(2)$.

13. $\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$.

To see this, set $y = \lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x$. The following shows $\ln(y) = 1 \Rightarrow y = e$

$$\ln(y) = \ln\left(\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x\right) = \lim_{x\to\infty}\frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} \overset{L}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1}{1 + \frac{1}{x}} = 1$$

14. $\lim_{x\to 0}\frac{\cos(x) - 1}{x^2} \overset{L}{=} \lim_{x\to 0}\frac{-\sin(x)}{2x} \overset{L}{=} \lim_{x\to 0}\frac{-\cos(x)}{2} = -\frac{1}{2}$.

15. $\lim_{x\to 0}\frac{\cosh(x) - 1}{x^2} \overset{L}{=} \lim_{x\to 0}\frac{\sinh(x)}{2x} \overset{L}{=} \lim_{x\to 0}\frac{\cosh(x)}{2} = \frac{1}{2}$.