Polynomial Approximation using inner products: Problems 1-6

Let a_1, a_2, a_3, a_4 be four degree four polynomial approximations of $f(t) = \exp\left(-\frac{t^2}{2}\right)$ obtained using the following four inner products:

$$\langle f, g\rangle_1 = \int_{-2}^{2} f(t)g(t)dt$$
$$\langle f, g\rangle_2 = f(-2)g(-2) + f(-1)g(-1) + f(0)g(0) + f(1)g(1) + f(2)g(2)$$
$$\langle f, g\rangle_3 = f(-2)g(-2) + f'(-2)g'(-2) + f(0)g(0) + f'(0)g'(0) + f(2)g(2) + f'(2)g'(2)$$
$$\langle f, g\rangle_4 = f(0)g(0) + f'(0)g'(0) + f''(0)g''(0) + f'''(0)g'''(0) + f''''(0)g''''(0)$$

Problem 1. How far is the approximation a_1 from the function f (where distance is measured using the $\langle\ ,\ \rangle_1$ inner product).

(a) 0.00408229

(b) 0.0215026

(c) 0.390543

(d) 0.419106

(e) 0.655499

Problem 2. True or False: $a_i(0) = 1$ for $i = 2, 3, 4$.

Problem 3. True or False: $a_i'(0) = 1$ for $i = 1, 2, 3, 4$.

Problem 4. Using the fact that $|f^{(5)}(t)| < 5.78306$, which of the following is the bound on the error $|a_4(t) - f(t)|$ for $-2 \le t \le 2$ given by Taylor's error formula?

(a) 0000717145

(b) 0.00114743

(c) 0.272616

(d) 1.54215

(e) 42.8696

Problem 5. Using the fact that $|f^{(5)}(t)| < 5.78306$, which of the following is the bound on the error $|a_2(t) - f(t)|$ for $-2 \leq t \leq 2$ given by the Lagrange error formula?

(a) 0000717145

(b) 0.0309371

(c) 0.175007

(d) 4.86494

(e) 24.3247

Problem 6. Which polynomial is orthogonal to each of $\{1, t, t^2, t^3, t^4\}$ using $\langle\ ,\ \rangle_1$?

(a) $512t - 332t^3 + 45t^5$

(b) t^5

(c) $(t-2)(t)(t+2)$

(d) $\frac{80}{7}t - \frac{40}{3}t^3 + 3t^5$

(e) $4t - 5t^3 + t^5$
