Problem set 2

Math 625 Spring 2024

April 21, 2024

1 Pagerank

Problem 1. Download the the file `facebook_combined.txt.gz` from
https://snap.stanford.edu/data/egonets-Facebook.html
and create a graph using the python command

```
G_fb = nx.read_edgelist("facebook_combined.txt", create_using = nx.Graph())
```

Compute the pagerank of the 4039 vertices of the resulting graph.

2 The power method

Problem 2. Let

$$A = \begin{bmatrix} 42 & 60 \\ -20 & -28 \end{bmatrix}, \qquad B = \begin{bmatrix} 4 & -8 \\ -6 & 18 \end{bmatrix}, \qquad C = \begin{bmatrix} 4 & -4 \\ -4 & 10 \end{bmatrix}.$$

(a) Find the eigenvalues and eigenvectors of A, B, and C exactly.

(b) Write a program:

Input: an nxn matrix A, a nonzero vector v, a natural number n

Output: An approximation of the eigenvalue of A having the largest absolute value and a corresponding normalized eigenvector.

```
Set w_0=v/||v||
For k=1 to n
Set v_k=A.w_(k-1)
Set l_k=<v_k,w_(k-1)}
Set w_k=v_k/||v_k||
Output: l_1,...l_n and w_n
```

Use the initial vector $v_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $n = 20$ and compare the output for each of these matrices. Pay attention to the rate of convergence.

Problem 3. Write some more programs:

(a) Input: an nxn matrix A, a real number s, a nonzero vector v, a natural number n

Output: an approximation of the eigenvalue of A-sI having the largest absolute value and a corresponding normalized eigenvector.

```
Set w_0=v/||v||
Set B=A-sI
For k=1 to n
Set v_k=B.w_(k-1)
Set l_k=<v_k,w_(k-1)}
Set w_k=v_k/||v_k||
```

(b) Input: an nxn matrix A, a real number s, a nonzero vector v, a natural number n

Output: an approximation of the eigenvalue of A closest to s and a corresponding normalized eigenvector.

```
Set w_0=v/||v||
Set B=A-sI
For k=1 to n
Solve Bv_k=w_(k-1)
Set l_k=1/<v_k,w_(k-1)}+s
Set w_k=v_k/||v_k||
```

(c) Use your programs, and good mathematical arguments, to find all the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 8 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$