On the Bateman-Horn conjecture for polynomials over large finite fields

We study the analogue of the classical Bateman-Horn conjecture on prime values of polynomials with the ring of integers replaced by the ring of polynomials over a large finite field. For non-associate, irreducible and separable polynomials $F_1, ..., F_m \in \mathbb{F}_q[t][x]$ and a natural number n we study the asymptotic of

$$\#\{f \in \mathbb{F}_q[t], \deg f = n | F_i(t, f) \text{ is irreducible}, 1 \leq i \leq m\}$$

for $m, n, \deg F_i$ fixed and $q \to \infty$. We obtain results for this problem which are much more general than known previously. We use the classification of finite simple groups to obtain some of the results.