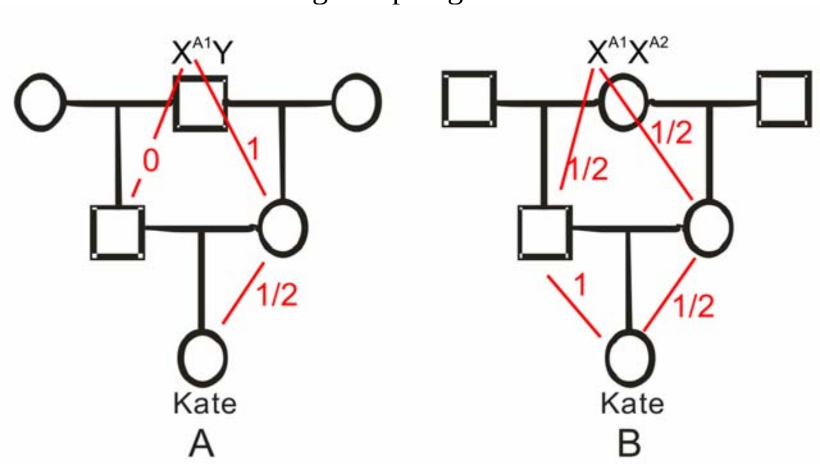
Problem Set 3 Name: ___________________

1. Consider the following two pedigrees:



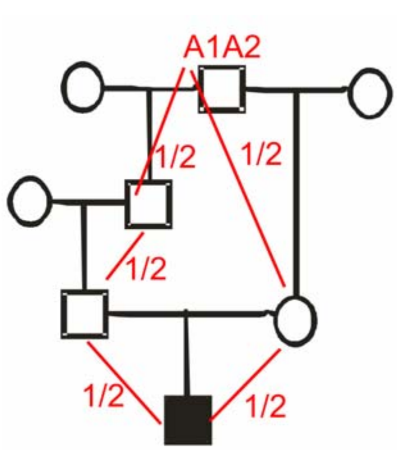
Ignore all ancestors that are not shared by Kate's parents

What are the probabilities of identity by descent in individual "Kate" for a randomly chosen X-linked allele in pedigrees A and B?

A: F=0, because Kate's father gets the Y chromosome from his father and not X^{A1}.
Kate cannot be X^{A1} X^{A1} homozygous.

B:
Probability that Kate is X^{A1} X^{A1} homozygous is ½ x ½ x ½ = 1/8
Probability that Kate is X^{A2} X^{A2} homozygous is ½ x ½ x ½ = 1/8
$F = P(X^{A1} X^{A1}) + P(X^{A2} X^{A2}) = 1/8 + 1/8 = 1/4$

2. Consider the pedigree below:



What is the pedigree inbreeding coefficient F of the male individual shown as a black square? Assume that all relevant genetic relationships are shown in the pedigree.

Probability that Johnny is A1A1 homozygous is ½ x ½ x ½ x ½ x ½ = 1/32
Probability that Johnny is A2A2 homozygous is ½ x ½ x ½ x ½ x ½ = 1/32

F = P(A1A1) + P(A2A2) = 1/16

3. Consider a fictional scenario:
The dominant **allele L** codes for lactase persistence. People who are recessive homozygotes (ll) cannot digest milk as adults; dominant homozygotes (LL) and heterozygotes (Ll) have no problem digesting milk. Among **200** people from Eastern Mongolia, **32** were unable to digest milk.
a. Assuming that the population does not deviate from a **Hardy-Weinberg equilibrium,** estimate the frequencies of the alleles **L** and **l**:

fr(ll) = 32/200 = 0.16 = q^2

fr(l)$_0$ = $\sqrt{q^2} = \sqrt{0.16} = 0.4$ fr(L)$_0$ = p = 1-q = 0.6

b. Milk in Mongolia is a common source of bacterial infection. Individuals who drink milk are very likely to contract a severe stomach infection and die. The mortality rate from stomach infection among milk drinkers (the LL and Ll genotypes) is 40%.

Using this information and the allele frequencies you have found above, fill in the table : (2 pt)

	LL	Ll	ll	total
frequency of each genotype	p^2=0.36	2pq =0.48	q^2=0.16	1
relative fitness (*w*)	0.6	0.6	1	
coefficient of selection (***s***)	0.4	0.4	1	
genotype frequencies after selection	$\frac{p^2 \times 0.6}{0.6p^2 + 0.6 \times 2pq + q^2} =$ $0.22/0.66 = 0.34$	$\frac{2pq \times 0.6}{0.6p^2 + 0.6 \times 2pq + q^2}$ $0.29/0.66 = 0.44$	0.16/0.66 = 0.24	1

c. Find the respective allele frequencies after selection:

fr(l)$_1$ = fr(ll) + 0.5fr(Ll) = 0.24+0.22 = 0.46 fr(L)$_1$ = 1-fr(l) = 0.54

d. What type of natural selection is operating on this population?:______________________
Directional selection for the recessive phenotype
f. What are the points of equilibrium for this type of natural selection? Which are stable and which are unstable? Why?
$\Delta q = q^2ps / (1-p^2s-2pqs)$; $\Delta q=0$ when q=0 or p=0;

when 0<q<1, $\Delta q > 0$ ➔ q is increasing → $\hat{q}=0$ is unstable, $\hat{q}=1$ is stable

4. The recessive allele c codes for cystic fibrosis. People who are recessive homozygotes (cc) have a tendency toward chronic lung infections. Mortality among people with cystic fibrosis is 15% greater than that of people without the disease. Dominant homozygotes show no symptoms of the disease. In a randomly mating population, 9% of newborns were diagnosed with cystic fibrosis.

A. Estimate the frequency of allele c, assuming a Hardy-Weinberg equilibrium:

We assume that the population of newborns does not deviate from the Hardy-Weinberg expectations: Thus, $fr(c)_0 = q = \sqrt{q^2} = \sqrt{0.09} = 0.3$ $fr(C) = 1-0.3 = 0.7$

B. What type of natural selection is operating?: Directional selection for the dominant phenotype

C. Calculate allele frequencies in this case after one and after two generations:

$\Delta q = q_1 - q \Rightarrow q_1 = q + \Delta q$

In the case of directional selection for the dominant phenotype

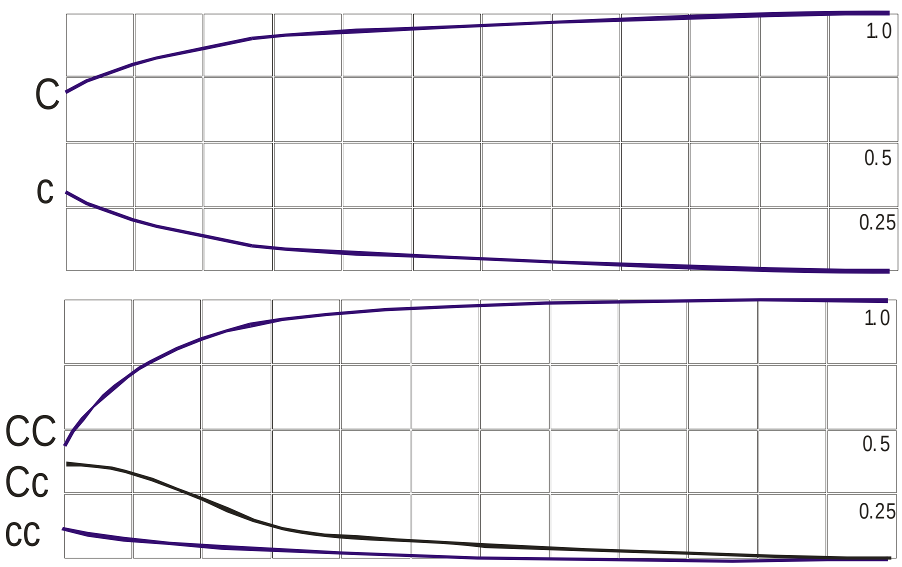
$\Delta q = -q^2ps/(1-q^2s)$ $[q^2=0.09, p=0.7, s=0.15]$

$\Delta q = -0.09*0.7*0.15/(1-0.09*0.15) = -0.0096$

$q_1 = q + \Delta q = 0.3 - 0.0096 =$ **0.29**

$q_2 = q_1 + \Delta q_1 = 0.29 - [0.29^2*0.71*0.15/(1-0.29^2*0.15)] = 0.29 - 0.009 =$ **0.28**

D. Predict how the allele and genotype frequencies in this case are likely to change through time. Make rough graphs:



5. For a certain type of balancing selection it was found that:

Δq = pq(0.6p - 0.4q)/(1- 0.6p^2-0.4q^2)

Find three points of equilibrium:
Which of them are stable and which of them are unstable?
For balancing selection, $\Delta q = pq(s_1p - s_2q)/(1 - p^2 s_1 - q^2 s_2)$ → $s_1=0.6$, $s_2=0.4$ →

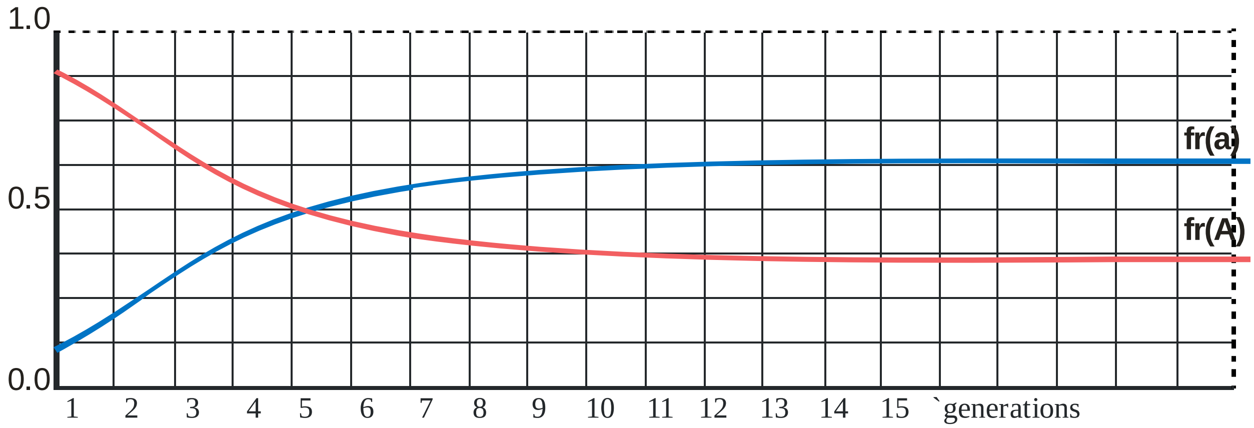
$\hat{q}_{st} = s_1/(s_1 + s_2) = 0.6/(0.6+0.4) = 0.6$

$\hat{q}$ =0 and 1 are unstable
What are the genotype frequencies at equilibrium?

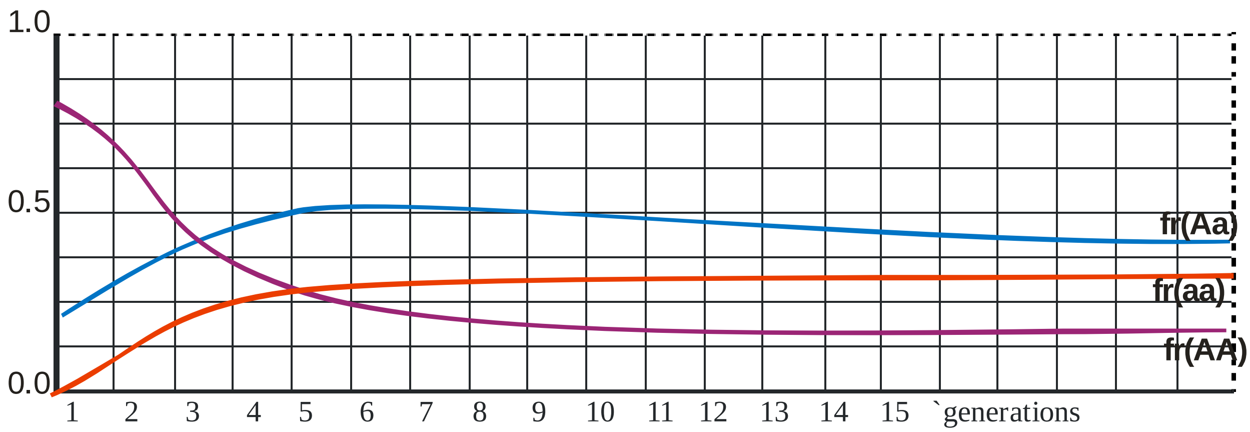
fr(AA) = _0.16_ fr(Aa) = _0.48_ fr(aa) = _0.36_

In the population of western Colorado, the frequency of allele **a** is 0.1.

Make an approximate chart showing how allele frequencies will change over the generations:



Make an approximate chart of the change in genotype frequencies over the generations:



6. Consider another fictional scenario:

Among a group of hunting and gathering native Australians, individuals who can detect a bitter taste in certain plants, even when the pertinent alkaloids are present in very low concentrations (**TT** homozygotes), are highly valued as plant gatherers. Non-tasters (**tt** homozygotes) don't even try to gather plants and pursue hunting instead. **Tt** heterozygotes try to gather plants, but often die from ingesting toxins. This situation results in the following values of relative fitness:

	TT	**Tt**	**tt**
W	1	0.2	0.8

a. What type of natural selection is operating? **Disruptive**

b. Find all points of equilibrium. Which of them are stable?

$s_1=1-w_1= 1-0.2=0.8$, $s_2=1-w_2 =1-0.8= 0.2$

$$\Delta p = \frac{p^2+(1-s_1)pq}{1-2pqs_1-q^2s_2} - p = \frac{p^2+(1-s_1)pq - p + 2p^2qs_1 + q^2ps_2}{1-2pqs_1-q^2s_2}$$

$$\Delta p = 0, when, p^2+(1-s_1)pq - p + 2p^2qs_1 + q^2ps_2 = 0$$

This simplifies:

$$pq(s_1 - 2s_1q + s_2q) = 0 \Rightarrow \Delta p = 0, when$$

$$p = 0, q = 0, or \quad s_1 - 2s_1q + s_2q = 0 \Rightarrow q = \frac{s_1}{2s_1 - s_2}$$

$$Three\ points\ of\ equilibrium: \hat{q} = 1, \hat{q} = 0, \hat{q} = \frac{0.8}{2\times 0.8 - 0.2} = 0.57$$

When $0<q<0.57$, $\Delta q<0$ ➔ q is declining

when $0.57<q<1$, $\Delta q>0$ ➔ q is growing ➔ $\hat{q} = 1\ is\ stable, \hat{q} = 0\ is\ stable, \hat{q} = 0.57\ is\ unstable$

c. The current frequency of allele **t** is 0.55. Make an approximate chart of allele frequency change in the tribe over generations.

fr(t)<0.57 ➔Δq<0➔fr(t) is going to decline