

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

**Final Examination
2 ½ Hours**

Mathematics 122

Fall 2006

Instructions:

Answer all questions.

Show all work.

1. Given $f(x) = \sqrt{x-4}$ and $g(x) = \frac{1}{x}$.
- Sketch the graph of f and determine its domain.
 - Use the graph of f to sketch the graph of f^{-1} .
 - Find an equation for f^{-1} .
 - Sketch the graph of $g(x)$ and determine its domain.
 - Use the graph of $g(x)$ as an aid in sketching the following. Label vertical and horizontal asymptotes.
 - $y = \frac{1}{x-4}$
 - $y = \left| \frac{1}{x-4} \right|$
 - Find $(g \circ f)(x)$.
 - Find the following and simplify:
 - $f(20)$
 - $g(x) + 2$
 - $\frac{g(x+h) - g(x)}{h}, h \neq 0$
2.
 - Construct Pascal's triangle for the binomial coefficients up to $n=5$.
 - Expand and simplify $(x-2y)^5$.
3. Sketch the graph of each of the following. Label vertices, centers, x-intercepts, y-intercepts and vertical and horizontal asymptotes where appropriate.
 - $y = 27 - x^3$
 - $x^2 + y^2 - 12x + 6y - 19 = 0$
 - $y = 16x - x^2$
 - $y = \sqrt{64 - x^2}$
 - $y = 2 + 5^{-x}$
 - $y = 3 \cos 2x, x \in [0, 2\pi]$
4.
 - Use the Factor Theorem to show that $(x-4)$ and $(x+2)$ are factors of $P(x) = x^4 - 5x^3 + 20x - 16$.
 - Determine all the zeros of $P(x) = x^4 - 5x^3 + 20x - 16$.
5. Given the points **A**(-3, -1), **B**(3, 3), **C**(-9, 8):
 - Find an equation of line AB.
 - Find the midpoint of AC.
 - Show that triangle ABC is a right triangle.

(over)

6. Solve each of the following for x :

a. $6^{x^2+x} = 36$

b. $2\cos^2 x - 5\sin x = 4, x \in [0, 2\pi]$

c. $\log_3(x+6) - \log_3(x+2) = \log_3 x$

7. Evaluate without use of a calculator:

a. $\cos\left(\frac{4\pi}{3}\right)$

b. $\tan^{-1}(1)$

c. $\sin 43^\circ \cos 17^\circ + \cos 43^\circ \sin 17^\circ$

d. $\log_4 6 + \log_4 2 - \log_4 3$

8. Verify each of the following trigonometric identities:

a. $\cot x + 1 = \csc x (\cos x + \sin x)$

b. $\frac{\cos x}{1 - \sin x} + \frac{\cos x}{1 + \sin x} = \frac{2}{\cos x}$

9. Let A be an angle in the second quadrant and B be an angle in the fourth quadrant.

If $\sin A = \frac{5}{13}$ and $\cos B = \frac{3}{5}$, find:

a. $\cos(A - B)$

b. $\tan A$

c. $\sin 2A$

d. $\tan \frac{B}{2}$

10. A rectangular field with an area of 2500 square feet is to be fenced in. Two opposite sides of the field will cost \$3 per foot of fencing; the remaining two sides cost \$2 per foot of fencing. Express the total cost of the fencing as a function of x , the length of one side of the field.