

QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS
Final Examination
2 ½ Hours

Mathematics 122

Fall 2007

Instructions:

Answer all questions.

Show all work.

1. Given $f(x) = \sqrt{x-2}$ and $g(x) = \frac{1}{x}$

- a. Sketch the graph of f and determine its domain.
- b. Use the graph of f to sketch the graph of f^{-1} .
- c. Find an equation for f^{-1} .
- d. Sketch the graph of g and determine its domain.
- e. Use the graph of $g(x) = \frac{1}{x}$ as an aid in sketching the following:

i) $y = \frac{1}{x+2}$

ii) $y = \left| \frac{1}{x} \right|$

- f. Find $(f \circ g)(x)$ and its domain.
- g. Find each of the following and simplify:

i) $g(-2)$

ii) $g(x+h)$

iii) $\frac{g(x+h) - g(x)}{h}, h \neq 0$

2. Let $P(2,0)$ and $Q(-5,12)$ be points in the plane.

- a. Find the distance between P and Q .
- b. Find the midpoint of the segment PQ .
- c. Find an equation of the line that passes through P and Q .
- d. Find an equation of the circle that passes through Q and has its center at P .

3. Sketch the graph of each of the following equations. Where appropriate, indicate center, vertices, x-intercepts, y-intercepts, and vertical and horizontal asymptotes.

a. $y = 2 \cos 2x, 0 \leq x \leq 2\pi$

d. $y = 1 + 4^{-x}$

b. $y = -x^2 + 2x - 3$

e. $y = 64 - x^3$

c. $y = |x - 7|$

f. $y = \sqrt{100 - x^2}$

4. a. Construct Pascal's triangle for the binomial coefficients up to $n=5$.

b. Expand and simplify $(3x - y)^5$.

5. Evaluate without using a calculator:

a. $\cos^{-1}\left(\frac{-1}{2}\right)$

c. $\tan \frac{5\pi}{6}$

b. $\cos 80^\circ \cos 50^\circ + \sin 80^\circ \sin 50^\circ$

d. $\log_2 96 - \log_2 6$

(continued on other side)

6. Solve for x .

a. $5^{x^2+x-2} = 25$

b. $\log_2(x+2) + \log_2(x-1) = 2$

c. $2\sin^2 x - \cos x - 1 = 0$ on the interval $[0, 2\pi]$

7. Verify the trigonometric identities:

a. $\frac{1 + \tan^2 x}{\tan x \csc^2 x} = \tan x$

b. $\cos^4 x - \sin^4 x = \cos 2x$

8. If $\cos A = -\frac{4}{5}$, where $\frac{\pi}{2} < A < \pi$ and $\tan B = \frac{5}{12}$ where $\pi < B < \frac{3\pi}{2}$, find

a. $\cot A$

b. $\cos(A - B)$

c. $\sin 2B$

d. $\cos \frac{A}{2}$

9. a. Determine which of the following are factors of $P(x) = x^4 - 9x^2 + 4x + 12$.
Explain your reasoning.

i) $x + 2$

ii) $x + 1$

iii) $x - 2$

b. Determine all the zeros of $P(x) = x^4 - 9x^2 + 4x + 12$.

10. An open box has a square base. If its surface area is 200 cm^2 , express its volume as a function of its base dimension x .