QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2 ½ Hours

Mathematics 122 **Fall 2008**

Instructions: **Answer all questions.** **Show all work.**

1. Given $f(x)=\sqrt{x-10}$ and $g(x)=\frac{1}{x}$.

a. Sketch the graph of f and determine its domain.

b. Use the graph of f to sketch the graph of f^{-1}.

c. Find an equation for f^{-1}.

d. Sketch the graph of $g(x)$ and determine its domain .

e. Use the graph of $g(x)$ as an aid in sketching the following:

1. $y=\frac{1}{x-10}$ **2.** $y=\left|\frac{1}{x-10}\right|$

f. Find $(g \circ f)(x)$.

g. Find the following and simplify:

1. $f(14)$ **2.** $g(x)-1$ **3.** $\frac{g(x+h)-g(x)}{h}$, $h\neq 0$

2. **a.** Construct Pascal's triangle for the binomial coefficients up to $n=5$.

b. Expand and simplify $(2x-y)^5$.

3. Sketch the graph of each of the following. Label vertices, centers, x-intercepts, y-intercepts and vertical and horizontal asymptotes where appropriate.

a. $y=216-x^3$ **b.** $x^2+y^2-2x+14y-31=0$

c. $y=6x-x^2$ **d.** $y=\sqrt{36-x^2}$

e. $y=6^{-x}+2$ **f.** $y=6\sin 2x$ on the interval $[0,2\pi]$

4. **a.** Use the Factor Theorem to show that $(x-5)$ and $(x+3)$ are factors of $P(x)=x^4-3x^3-15x^2+19x+30$.

b. Determine all the zeros of $P(x)=x^4-3x^3-15x^2+19x+30$.

5. **a.** Given two points (-1, 2) and (2, 8), find the slope of line l_1 passing through the two points.

b. Find an equation of the line l_2 passing through (4, 2) that is perpendicular to l_1.

c. Find an equation of the circle having a diameter with endpoints (-1,2) and (2,8) .

(over)

6. Given that A is an angle in the second quadrant and B is an angle in fourth quadrant. If $\sin A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, find:

a. $\cos(A - B)$

b. $\tan B$

c. $\sin 2A$

d. $\cos \frac{B}{2}$

7. Prove the trigonometric identities:

a. $\cot x + \tan x = \csc x \sec x$

b. $\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x} = 2\sec x$

8. Evaluate:

a. $\sin \frac{5\pi}{4}$

b. $\tan(\sin^{-1}\left(\frac{1}{2}\right))$

c. $\sin 36° \cos 24° + \cos 36° \sin 24°$

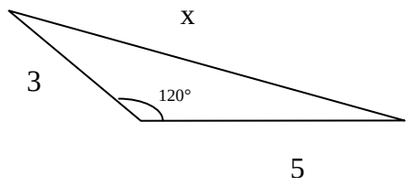
9. Solve for x:

a. $\log_4 x + \log_4 (x+2) = \frac{3}{2}$

b. $8^{x^2+x-2} = 64^x$

c. $2\sin^2 x = \cos x + 1$ on the interval $[0, 2\pi]$

d.



10. Megan and Hayim want to fence in a rectangular vegetable garden and subdivide it into three regions by using two additional sections of fence parallel to one side, x, of the rectangle. The total enclosed area is to be 2000 ft.^{2}. Express the total length of the fencing as a function of x.