

QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS

Final Examination  
2 ½ Hours

Mathematics 122

Spring 2007

**Instructions:** Answer all questions. Show all work.

- 1) a) Find the center and radius of the circle  
 $x^2 + y^2 - 2x + 8y - 19 = 0$ .
- b) Given the points A (4, -1) and B (-2, -3), write an equation of the circle in which AB is a diameter.
- 2) a) Using the calculator, find all real solutions of  $2x^4 + 4x = 3x^2 + 2$  to the nearest hundredth.
- b) Using the calculator, find the local maximum and local minimum values of  $f(x) = x^3 - x^2 - 2x$  to the nearest hundredth.
- 3) Expand and simplify  $(3y + 2)^5$ .
- 4) Let  $f(x) = \sqrt{x-3}$  and  $g(x) = 2x^2 + 5$ .
- a) Sketch the graph of  $f$  and determine the domain and range of  $f$ .
- b) Using the result of part (a), sketch the graph of  $f^{-1}$ . Then determine the domain and range of  $f^{-1}$ .
- c) Write an equation for  $f^{-1}$ .
- d) Find  $(g \circ f)(x)$ .
- e) Find and simplify  $\frac{g(x+h) - g(x)}{h}$ ,  $h \neq 0$ .
- 5) What two numbers whose sum is 104 will yield a maximum product? What is the maximum product? Find a function that models the questions and use that model to answer the questions.
- 6) Sketch the graph of each of the following. Label vertices,  $x$  and  $y$  intercepts, and vertical and horizontal asymptotes where appropriate.
- a)  $y = x^2 - 8x$
- b)  $y = 4 \cos 2x$  in the interval  $[0, 2\pi]$
- c)  $y = x^3 + 125$
- d)  $y = 4^{-x} + 1$
- e)  $y = \frac{1}{x+3}$

(over)

7) Find the exact value of each of the following without a calculator.

a)  $\cos \frac{5\pi}{4}$

b)  $\sin^{-1} \left( -\frac{\sqrt{3}}{2} \right)$

c)  $\log_4 8 + \log_4 6 - \log_4 3$

d)  $\cos 70^\circ \cos 40^\circ + \sin 70^\circ \sin 40^\circ$

8) Let  $P(x) = 3x^4 - 11x^3 - x^2 + 19x + 6$ .

a) List all possible rational zeros of  $P$ .

b) Find all the real zeros of  $P$  algebraically.

9) Solve for  $x$ :

a)  $4^{4x} = 8^{2x+6}$

b)  $\log_4(x+3) + \log_4(2-x) = 1$

c)  $3\cos x + 3 = 2\sin^2 x$ , where  $x$  is in the interval  $[0, 2\pi]$

10) a) Prove the identity:  $\sin x \tan x + \cos x = \sec x$ .

b) In  $\triangle ABC$ ,  $BC = 4$ ,  $\angle A = 40^\circ$ , and  $\angle C = 60^\circ$ . Find  $AC$  to the nearest hundredth.

11)

If  $\cos A = -\frac{24}{25}$  with  $\angle A$  in quadrant II and  $\tan B = \frac{8}{15}$  with  $\angle B$  in quadrant III, find:

a)  $\csc A$

b)  $\sin(A + B)$

c)  $\cos 2A$

d)  $\sin \frac{A}{2}$