QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 Hours
Math 131 **Spring 2008**

Please answer all ten questions in the blue books provided, and show all your work.

1. Evaluate the following limits. For parts (a)-(c), give algebraic reasons why the limit has the value you give. In part (d), you need only compute the limit, without giving reasons. (If a limit is infinite, say so. If the limit is neither infinite nor equal to any real number, state that it does not exist, and give a reason why.)

   (a) $\lim_{x\to-2}\frac{x^2+x+3}{x+2}$.

   (b) $\lim_{x\to-2}\frac{x^2+5x+6}{x^2+x-2}$.

   (c) $\lim_{x\to3}(2x^2+\sqrt{x-3})$.

   (d) $\lim_{x\to+\infty}\frac{0.003x^3+200x^2-4x+1}{0.001x^3-500x^2+10x}$.

2. Consider the function:

$$g(x) = \begin{cases} 1+\sqrt{x}, & \text{if } 0 \leq x \leq 1 \\ e^x, & \text{for all other } x \end{cases}$$

   (a) State whether g is continuous at $x = 1$, and also at $x = 1$. Give reasons for your answers.

   (b) Is g continuous on the closed interval $[1, 5]$?

   (c) Is g continuous on the closed interval $[0, 1]$?

   (d) Name all points in the domain of g at which g is not differentiable.

3. The price $p(t)$ of a particular model of car changes over the course of a year according to the function $p(t) = 15,000 - 1000(t - 0.6)^2$ as t ranges from 0 (the start of the year) to 1 (the end of the year).

   (a) Find the average rate of change of the price over the first half of the year (that is, from time $t = 0$ to time $t = 0.5$).

   (b) Find the instantaneous rate of change of the price at time $t = 0.3$.

   (c) At what value of t is the tangent line to the curve $y = p(t)$ exactly horizontal? What is the price of the car at this time? Is this the best time for you to buy a car, or the worst, or neither?

4. Give the derivatives of the following functions. Express each answer in reasonably simple form. (In part (f), your answer may involve y as well as x.)

   (a) $f(x) = 2x^4 - \pi x^2 + 17x - \frac{1}{x}$.

   (b) $f(x) = 4x^3\sqrt{x-4}$.

   (c) $f(x) = \frac{3x^2-1}{4x+1}$.

   (d) $f(x) = \ln(1 + e^{(x^2+3x)})$.

   (e) $f(x) = \frac{x^{\frac{1}{3}} \cdot \sqrt{4x^2+7}}{(x-6)^{17}}$. (Suggestion: logarithmic differentiation.)

   (f) The function defined by the equation $y^3 = 2xy^2 + 2x^2y - 7$.

5. Let $h(x)$ be the function $x^3 - 6x^2 + 9x + 12$. For each of the following questions, show your work or give reasons for your answers.

   (a) Find the x-coordinates of all relative maxima of h.

   (b) Find the x-coordinates of all relative minima of h.

   (c) Determine whether h has an absolute maximum and/or an absolute minimum on the interval $[0, 5]$, and if so, state what each of them is and at which x-coordinates they occur.

6. Sketch the graph of a function g satisfying the following conditions:

   - The domain of g is the set of all real numbers except 0, and its second derivative g'' exists and is continuous on this entire domain.
   - g is symmetric with respect to the y-axis.
   - $\lim_{x \to 0} g(x) = 4$.
   - g is increasing on the intervals $(-\infty, -3)$ and $(0, 3)$, and decreasing on $(-3, 0)$ and $(3, +\infty)$.
   - g has inflection points at $x = \pm 4$.
   - g has no asymptotes.
   - $g(3) = 6$, $g(4) = 3$, and g has x-intercepts at $x = \pm 5$.

7. A boat is pulled in to a dock by means of a rope with one end attached to the bow of the boat, the other end passing through a ring atttached to the dock at a point 6 feet higher than the bow of the boat. If the rope is pulled in at the rate of 2 ft./sec., how fast is the boat approaching the dock when there are 10 feet of rope between the ring and the bow?

8. Using the definition of the derivative, find the slope of the tangent line to the curve $y = \sqrt{x}$ at the point $(9, 3)$, and find an equation for this tangent line. Please write out your solution algebraically, as a limit, and evaluate the limit, without referring to a calculator solution or applying any formulas for differentiation.

9. Find all intervals of increase and decrease, and all intervals of upward and downward concavity, of the function:
$$f(x) = \frac{4x^3 + x - 5}{x}.$$

10. You open a bank account by depositing \$1000. The annual interest rate on the account is 4%, and you make no further withdrawals or deposits.

    (a) Assuming interest is compounded monthly, find a formula for the amount in the account at the end of t years.

    (b) How many years will it take before the amount in the account reaches \$2000?

    (c) Find a formula for the rate of change of the amount in the account (in dollars per year) at time t years.