Department of Mathematics, Queens College
Math 141 Final Exam, Fall 2008

The exam has two parts. You have $2\frac{1}{2}$ hours to answer the questions.

PART I [30 POINTS]. CLEARLY WRITE THE LETTER OF THE CORRECT ANSWER IN YOUR EXAM BOOK. YOU MUST SHOW HOW YOU ARRIVE AT YOUR DECISION.

1. $\lim_{x\to 0} \dfrac{1-\sqrt{1-4x^2}}{x^2}$

(A) is 1 (B) is 2 (C) is 0 (D) does not exist

2. $\lim_{x\to -3^+} \dfrac{|x+1|}{x^2-9}$

(A) is 0 (B) is ∞ (C) is $-\infty$ (D) does not exist

3. $\lim_{x\to\infty} \dfrac{2x^2+x+10}{x^3+5x^2-x}$ is

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{2}{5}$ (D) $\frac{5}{2}$

4. If the function $f(x) = \begin{cases} \dfrac{\sin(ax)}{x} & x \neq 0 \\ 4a+1 & x = 0 \end{cases}$ is continuous at $x = 0$, then a is

(A) $-\frac{1}{3}$ (B) $\frac{1}{3}$ (C) -3 (D) 3

5. If the line tangent to the graph of $f(x) = x^3 + 1$ at the point $(a, f(a))$ passes through the origin, then a is

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\sqrt[3]{2}$ (D) $\frac{1}{\sqrt[3]{2}}$

6. If $f(x) = x\,|x|$, then

(A) f is differentiable everywhere.
(B) f is differentiable everywhere except at $x = 0$.
(C) f is continuous everywhere except at $x = 0$.
(D) both (B) and (C) are correct.

7. If ε is small, the best linear approximation to the volume of a cube of side-length $10+\varepsilon$ is

(A) 1000 (B) $1000+100\varepsilon$ (C) $1000+200\varepsilon$ (D) $1000+300\varepsilon$

8. My calculator suggests that the global (absolute) maximum of $f(x) = \dfrac{x\cos x}{x^2+1}$ on $(-\infty, \infty)$ is about

(A) 0.362 (B) 0.364 (C) 0.366 (D) 0.368

9. The position of an object moving along a straight line is given by $s = t^2 \cos t$, where s is measured in feet and t is the time measured in seconds. The initial acceleration of the object in $\mathrm{ft}/\mathrm{sec}^2$ is

(A) 0 (B) 1 (C) -1 (D) 2

10. If $f'(x) = (x+1)(x-1)^2$, then f has

(A) a local minimum and a local maximum
(B) a local maximum only
(C) a local minimum and a critical point of neither type
(D) a local maximum and a critical point of neither type

continued on the other side $\longrightarrow$

PART II [70 POINTS]. SOLVE THE FOLLOWING 5 PROBLEMS. SHOW ALL YOUR WORK.

Problem 1. [12 points] Use the Intermediate Value Theorem to show that the equation

$$\sqrt{x} + \sqrt{x+1} = 4$$

has a positive solution. Then use your calculator to estimate the numerical value of this solution (there is only one!) to three decimal places.

Problem 2. [16 points] In each case, find the derivative $y' = \frac{dy}{dx}$:

(i) $y = \left(\frac{x}{x^4+1}\right)^2$ (ii) $y = \sqrt{x + \sqrt{x}}$

(iii) $y = (x+1)^3 \tan(5x)$ (iv) $y^3 + \sin(xy) = x^2$

Problem 3. [12 points] The base radius of a cone is expanding at the rate of 1 in/min while its height is shrinking at the rate of 3 in/min. How fast is the volume of this cone changing at the moment when the base radius is 30 in and the height is 40 in? Is the volume increasing or decreasing at that moment? (Hint: The volume V of a cone of base radius r and height h is given by the formula $V = \frac{\pi}{3} r^2 h$.)

Problem 4. [15 points] Consider the function $f(x) = \frac{x}{x^2+4}$.

(i) Find the domain of f and check for possible vertical and horizontal asymptotes of its graph.

(ii) Find the formulas for f' and f'' and use them to determine the intervals of increase/decrease and concavity of f. Make sure you clearly identify the critical point(s) and their type (i.e., local max, local min, neither) as well as the inflection point(s).

(iii) Using your findings in (i) and (ii), sketch the graph of f.

Problem 5. [15 points] A trapezoid is inscribed in a semi-circle of radius 2, as shown.

(i) Find the height h in terms of the length x shown in the figure. Use this to express the area $A = (x+2)h$ of the trapezoid as a function of x only.

(ii) Using calculus, find the value of x which maximizes A on the interval $0 \le x \le 2$.

(iii) What is the largest area that such an inscribed trapezoid can have?