

**QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS**

**Final Examination  
2½ Hours**

**Mathematics 141**

**Spring 2008**

**Instructions:**

**Answer all questions.**

**Show all work.**

1) a) Using algebraic methods, either compute the limit or determine that it is  $\infty$  or  $-\infty$  :

i)  $\lim_{x \rightarrow 3^+} \frac{\frac{1}{2} - \frac{1}{x-1}}{x^2 - 3x}$

ii)  $\lim_{x \rightarrow 3^+} \frac{x^2 - 7x + 12}{x^2 - 6x + 9}$

b) Using your calculator's table feature, estimate  $\lim_{x \rightarrow 3^-} \frac{2^x - 8}{x - 3}$  to 3 decimal places. Enter suitable lines of your table into your exam book.

2) a) Give the definition that a function  $f$  is continuous at a point  $x = a$

b) Let  $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ x + 2 & \text{if } 0 < x < 2 \\ 8 - x^2 & \text{if } x > 2 \end{cases}$

Using only the definition in a), determine whether  $f$  is continuous  
i) at  $x = 0$                       ii) at  $x = 2$

c) Sketch the graph of  $f$ .  
d) Find the range of  $f$ .

3) a) Carefully state the Intermediate Value Theorem.

b) Use the Intermediate Value Theorem to show that there is a negative number  $c$  such that  $c^4 - 1 = 30\sqrt{2 - c}$

c) Use your calculator to approximate  $c$  to 3 decimal places.

4) a) Give the definition of  $f'(x)$ , the derivative of a function  $f$ .

b) Using only the definition in part a), find  $f'(x)$  if  $f(x) = \sqrt{2x + 3}$ .

5) a) Find the derivative,  $\frac{dy}{dx}$ :

i)  $\frac{3}{x^4} + \cos x \sqrt[3]{4x + 5}$

ii)  $y = \frac{\tan(x^2 + 1)}{\csc 2x}$

b) Find all points on the curve  $y = (x^2 - 121)^3 (x^2 - 25)$  at which the tangent line is horizontal.

6) Let  $C$  be the curve described by the equation  $x^2 y + x y^4 = 2x - 12$ .

a) Find an equation for the tangent line to  $C$  at the point  $(3, -1)$ .

b) Suppose  $f$  is a function with  $y = f(x)$  for  $(x, y)$  on  $C$  near  $(3, -1)$ . Use linear approximation to estimate  $f(2.95)$

(over)

7) Let  $f(x) = x^4 - 4x^3 + 12$

- a) On which intervals is  $f$  increasing? decreasing?
- b) On which intervals is the graph of  $f$  concave up? concave down?
- c) Draw the graph of  $f$ , labeling local extrema and inflection points, if any.

8) Suppose that a single function  $f$  has all of the following properties:

$$f'(x) > 0 \text{ if } 0 < x < 5 \text{ or } x > 5, \quad f'(x) < 0 \text{ if } x < 0,$$

$$f''(x) > 0 \text{ if } 3 < x < 5, \quad f''(x) < 0 \text{ if } x < 0, \quad 0 < x < 3, \text{ or } x > 5,$$

$$f(0) = 0, \quad f(3) = 2,$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty, \quad \lim_{x \rightarrow \infty} f(x) = 2,$$

$$\lim_{x \rightarrow 5^-} f(x) = \infty, \quad \lim_{x \rightarrow 5^+} f(x) = -\infty$$

Draw the graph of  $f$  using this information. Label local extrema and inflection points, if any.

9) A rectangular plot is to be fenced on 3 sides with metal fencing which costs \$2/ft and on the fourth side with wooden fencing which costs \$3/ft. The area of the plot is to be 50,000 sq. ft.

- a) What is the minimum cost for the project?
- b) What is the minimum cost for the project if only 100 ft of the wooden fencing is available?