

**QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS**

**Final Examination  
2.5 Hours**

**Mathematics 142**

**Fall 2008**

**Instructions:**

**Answer all questions.**

**Show all work.**

1.(8) Use the definition of the definite integral (i.e., the limit of the Riemann sum) to evaluate  $\int_2^3 x^2 dx$ .

$$\left[ \text{Note: } \sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \right].$$

2.(7) Find the exact value of:  $\int_{-5}^5 \sqrt{25-x^2} dx$  without using your calculator.  
(Hint: Consider the geometric significance of the definite integral.)

3.(20) Find  $\frac{dy}{dx}$  for each of the following:

a)  $y = \frac{x^3 \sqrt{x^2+1}}{(x+3)^{3/2}}$        $\left[ \text{Hint: Use logarithmic differentiation.} \right]$       b)  $y = \sin^{-1}(e^x)$       c)  $y = \ln(x^3) + (\ln x)^3$

d)  $y = x^x + x^3 + 3^x + 3^3$       e)  $y = \int_{\sin x}^3 \sqrt{t^3+1} dt$

4.(36) Evaluate each of the following integrals:

a)  $\int \frac{x dx}{\sqrt{4-x^2}}$       b)  $\int_0^{1/2} \frac{dx}{1+4x^2}$  (give an exact answer.)

c)  $\int \frac{1+2^{2x}}{2^x} dx$       d)  $\int \frac{1}{x\sqrt{1-(\ln x)^2}} dx$

5.(18) Let R be the region in the plane bounded by the curves  $y=9-x^2$  and  $y=3-x$ . Set up, but do not evaluate, the definite integrals to compute

- a) the area of R
- b) the volume of the solid generated by rotating R about the  $x$ -axis
- c) the volume of the solid generated by rotating R about the line  $x=-2$ .

6.(8) Let  $f(x) = \frac{1}{x+1}$  for  $x \geq 0$ .

- a) Show that  $f$  has an inverse. Call it  $g$ .
- b) Find the domain of  $g$  and sketch the graph of  $y=g(x)$ .
- c) Find  $g'(1/2)$  in two ways:
  - i. by using the general formula for the derivative of an inverse function
  - ii. by finding an expression for  $g(x)$  and then differentiating it.

7.(8) Assuming that the decay rate of a certain radioactive substance is proportional to its mass and that 100 gm of this material decays to 80 gm in 17 hours, find its half-life.