

**QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS**

**Final Examination
2.5 Hours**

Mathematics 143

Fall 2008

Instructions:

Answer all questions.

Show all work.

1) Without the use of a calculator, find the exact value of $\lim_{x \rightarrow \infty} (1+2x)^{\ln 2 / \ln x}$.

2) Solve the following differential equation for y :

$$\frac{dy}{dx} = x(y^2 + 1) \quad \text{given that } y(1) = 2.$$

3) Integrate each of the following:

a. $\int e^x \sin x dx$

e. $\int \frac{\sin^2 \sqrt{x}}{\sqrt{x}} dx$

b. $\int \frac{\sqrt{x}}{x+1} dx$

f. $\int \frac{\sqrt{4+x^2}}{x} dx$

c. $\int \frac{3x^2 + 4x + 7}{x(x^2 + 1)} dx$

g. $\int \sec^3 x \tan x dx$

d. $\int \frac{x+3}{x^2 + 2x + 2} dx$

4) Determine the convergence or divergence of the following improper integrals. If an integral converges, find its value.

a. $\int_0^{\infty} \frac{e^x}{e^{x+1}} dx$

b. $\int_{-1}^1 \frac{1}{x^3} dx$

5) Without the use of a calculator, find the limits of the following sequences. If a sequence diverges, say so.

a. $\left\{ \frac{m!m^2}{(m+2)!} \right\}$

b. $\left\{ \frac{e^{2n} - 1}{n^2} \right\}$

6) Determine if each of the following series converges or diverges. Justify your answer.

a. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

b. $\sum_{n=1}^{\infty} \frac{(-1)^n \sin n}{n^2 + 1}$

c. $\sum_{n=1}^{\infty} \frac{n^2 + n + 1}{n^4 - n^2 + 1}$

d. $\sum_{n=1}^{\infty} \frac{(-1)^n (2n)!(e^n)}{(2n+1)!}$

7) Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-3)^n n}{4n+3}$.

8) a) Write the third Taylor polynomial for $f(x) = \sqrt[3]{x}$ at $a=8$.

b) If we use the polynomial computed in (a) to estimate $f(x)$ on the interval $[8, 8.2]$, estimate the maximum error that can occur.