

**QUEENS COLLEGE  
DEPARTMENT OF MATHEMATICS**

**Final Examination  
2½ Hours**

**Math 143**

**Spring 2008**

**Instructions:**

**Answer all questions.**

**Show all work.**

1. Find the following limits ( $+\infty$  and  $-\infty$  are allowed):

a.  $\lim_{x \rightarrow 0^+} \frac{e^x + e^{-x} - 2}{x^3}$

b.  $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - x)$

c.  $\lim_{x \rightarrow 0} (1+3x)^{\frac{2}{x}}$

2. Compute the following integrals:

a.  $\int_0^1 x e^{3x} dx$  (exact value required.)

b.  $\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$

c.  $\int \sec^4 x dx$

d.  $\int \frac{2x+4}{x^2(x-2)} dx$

3. Find the limits of the following sequences:

a.  $\left\{ \frac{\tan^{-1}(n^2)}{\sqrt{n}} \right\}_{n=1}^{\infty}$

b.  $\left\{ \frac{(n!)^2}{(2n)!} \right\}_{n=1}^{\infty}$

c.  $\left\{ \int_e^n \frac{dx}{x(\ln x)^2} \right\}_{n=3}^{\infty}$

4. Test for convergence (absolute/conditional) or divergence:

a.  $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^2 + 3n + 5}}$

b.  $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n!}}$

c.  $\sum_{n=0}^{\infty} \frac{3^{2n+3}}{2^{3n+2}}$

5. Find the exact interval of convergence for the following:

a.  $\sum_{n=1}^{\infty} \frac{x^{2n}}{n 2^n}$

b.  $\sum_{n=1}^{\infty} \frac{(3x+2)^n}{\sqrt[3]{n!}}$

(over)

6. a. Use your calculator to compute  $\sum_{k=0}^{100} \frac{(-1)^k}{k^4 + k^2 + 1}$ .

b. Using the value in part a above,

estimate the infinite series  $\sum_{k=0}^{\infty} \frac{(-1)^k}{k^4 + k^2 + 1}$ .

7. Use your calculator to compute the length of the curve  $y=x^3 - x$  between  $x=-1$  and  $x=1$ .

8. a. Find the Maclaurin series for  $f(x)=\cos(x^3)$ .

b. Using the result of part a, compute  $\int_0^1 \cos(x^3) dx$  with an error of  $< .001$ .