QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2.5 Hours

Mathematics 151 **Fall 2008**

Instructions: **Answer all questions.** **Show all work.**

1) Let $f(x)=\begin{cases}\frac{100x-x^3}{10+x} & \text{if } x\neq\pm 10\\ 0 & \text{if } x=-10\end{cases}$

(a) Find the domain of f.

(b) Sketch the graph of $y=f(x)$.

(c) Find the range of f.

(d) Find all values of x where f is discontinuous. Use a suitable computation to justify each answer.

2) (a) Use methods of calculus (not your calculator) to find each of the following limits. If a limit is $+\infty$, $-\infty$, or does not exist, explain why.

(*i*) $\lim\limits_{x\to -2}\ \dfrac{x^2+3x+2}{3x^2+6x}$ **(*ii*)** $\lim\limits_{x\to -1^-}\ \dfrac{x}{x^2-1}$

(*iii*) $\lim\limits_{x\to 0}\ \dfrac{\sin^{10}x\cos x}{x^{11}+4x^{10}}$ **(*iv*)** $\lim\limits_{x\to 0}\ f(x)$, where $f(x)=\begin{cases}\sqrt{9-x} & \text{if } x<0\\ -3 & \text{if } x=0\\ \frac{6}{2-x} & \text{if } 0<x<2\end{cases}$

(b) **USE YOUR CALCULATOR** to find any and all horizontal asymptotes of the graph of

$$y=F(x)=\left(\frac{x+1}{x+2}\right)^x$$

Be sure to copy the tables that you construct into your exam booklet to justify your conclusions.

3) Let $g(x)=\dfrac{9}{2x-1}$

(a) **Use the definition of the derivative** to find $g'(x)$.

(b) Find all points on the graph of *g* at which the tangent line to the graph is parallel to the line with equation $y=2x+1$.

4) An object moves along a horizontal line so that its position, *s*, at any time $t\geq 0$ is given by $s=t^4-12t^3+48t^2-68t+30$, where *s* is in feet and *t* is in seconds. Find the first and last times that its acceleration is equal to zero. Then find the displacement of the object between these two times.

5) (a) In each of the following, find $\dfrac{dy}{dx}$ (algebraic simplification is not necessary.)

(*i*) $y=\sqrt[8]{x^8+x^{-8}}$ **(*ii*)** $y=\dfrac{\cot(3x+4)}{\csc(4x+3)}$ **(*iii*)** $x^3y+xy^3-\sin x+\cos y=1$

(b) Use differentials (linear approximation) to estimate $\sqrt[4]{9999}$.

(continued on other side)

6) Answer <u>either</u> (a) or (b).

(a) Use the Intermediate Value Theorem to show that the equation $\sin x = 1 - x$ has a solution on the closed interval $[0, 2]$. Then **<u>USE YOUR CALCULATOR</u>** to estimate this solution, accurate to five decimal places.

or

(b) Let f be a differentiable function on the closed interval $[0, 4]$. If $f(4) = 10$ and if $2 \le f'(x) \le 5$ for all x between 0 and 4, use the Mean Value Theorem to find the smallest and largest possible values of $f(0)$.

7) (a) A large ice block is melting in such a way that it is always a cube. If the surface area of the block is decreasing at the rate of 72 sq.in/min. when its edge measures 3 inches, how fast is the volume of the block changing at that instant? **<u>Interpret your answer.</u>**

(b) If $h(x) = \sin 2x$, compute $h^{(21)}\left(\frac{\pi}{6}\right)$.

8) Let $y = f(x) = x^4 - 8x^3 + 18x^2 - 11$.

(a) Answer the following questions concerning the graph of f.

(*i*) On what interval(s) is it increasing? decreasing?

(*ii*) Does it have any relative extrema? If so, find the x-coordinate of each and classify each as a relative maximum or a relative minimum.

(*iii*) On what interval(s) is it concave upward? concave downward?

(*iv*) Does it have any points of inflection? If so, find the x-coordinate of each.

(b) Using the information found in part (a), sketch the graph of $y = f(x)$.

9) A rectangular box with a volume of 2000 cu. in. has a square top and bottom but is open in the front. The material for the top and bottom panels costs 4¢/sq.in., the material for the side panels costs 3¢/sq.in., and the material for the back panel costs 2¢/sq.in. Find the minimal cost of constructing such a box.

10) (a) Use a limit of a Riemann sum to compute the area of the region in the first quadrant bounded by the graph of $y = 16 - x^2$ and the coordinate axes.

$$\left(\text{Note: } \sum_{i=1}^{n} 1 = n;\ \sum_{i=1}^{n} i = \frac{n(n+1)}{2};\ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}. \right)$$

(b) Find each of the following indefinite integrals.

(*i*) $\displaystyle\int \frac{dx}{\sqrt{x}\left(2+\sqrt{x}\right)^2}$

(*ii*) $\displaystyle\int \frac{\tan^2\left(\frac{1}{x}\right)\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$