QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2½ Hours

Mathematics 151 **Spring 2008**

Instructions: **Answer all questions.** **Show all work.**

1) Use analytical methods (not your calculator) to find each of the following limits. If the limit is $+\infty$ or $-\infty$ or does not exist, explain why.

a) $\lim\limits_{x \to \infty} \dfrac{x^3+6x}{3x^3-x^2+1}$

b) $\lim\limits_{x \to 8} \dfrac{\sqrt{x+1}-3}{x-8}$

c) $\lim\limits_{x \to -7} \dfrac{3x+21}{|x+7|}$

d) $\lim\limits_{x \to 3^-} \dfrac{x^2-7}{x-3}$

2) The function $f(x) = \dfrac{2x^4-2}{\sqrt[3]{x}-1}$ is not defined when $x=1$.

a) Use a suitable table of values to estimate $\lim\limits_{x \to 1} f(x)$ to five decimal places. Be sure to copy the table into your answer booklet.

b) State the formal definition of continuity of a function f at a number $x=a$.

c) Can $f(1)$ be defined to make the new function continuous at $x=1$? Explain.

3) Use the definition of the derivative to find $f'(x)$ when $f(x) = \dfrac{1}{1-3x}$.

4) Find $\dfrac{dy}{dx}$ for each of the following. Do not simplify.

a) $y=6x^4-4\sqrt{x}+\dfrac{2}{\sqrt{5x}}+\pi^2$

b) $x^3y^2-2x^3+y^5=6$

c) $y=\sqrt{\dfrac{\cos x}{2+\sin x}}$

d) $y=\cos\left(\tan\sqrt{\cos x}\right)$

5) A cone-shaped paper cup is being filled with water at the rate of 3 cubic centimeters per second. If the height of the cup is 10 centimeters and the radius of its base is 6 centimeters, how fast is the water level rising when the level is 5 centimeters?

$\left(V=\dfrac{1}{3}\pi r^2 h\right)$

6) a) Show that $y=f(x)=x^5+3x-2$ has exactly one real root. Be sure to explain.

b) Using a graphing calculator, estimate the real zero of x^5+3x-2 to three decimal places.

7) Let $y=f(x)=\dfrac{2x^2}{x^2-25}$

a) Find the domain of f and any and all vertical and horizontal asymptotes of its graph.

b) Find the formulas for f' and f'' and use them to determine intervals of increase/decrease and concavity of f. Make sure you clearly identify the critical point(s) and their type (i.e., local max, local min, neither) as well as the inflection point(s).

c) Use your findings in **a)** and **b)** to sketch the graph of f.

8) A box with a square base and a closed top has a volume of 3,000 cubic feet. The top costs \$4/sq. ft, the bottom costs \$8/sq. ft, and the sides cost \$2/sq. ft. Find the dimensions of the box that will minimize its total cost of construction.

9) Find the integral as a limit of Riemann sums:

$$\int_0^3 \left(x^2-1\right)dx$$

$$\left[\text{Note: } \sum_{i=1}^{n} i=\frac{n(n+1)}{2},\ \sum_{i=1}^{n} i^2=\frac{n(n+1)(2n+1)}{6}\right]$$

10) Integrate:

a) $\displaystyle\int \frac{6x^2-4x+5}{\sqrt{x}}\,dx$

b) $\displaystyle\int \frac{\cos 6x}{(1+\sin 6x)^3}\,dx$