Queens College
Department of Mathematics

Draft Final Examination
2-1/2 Hours

Math 152 **Fall 2008**

ANSWER ALL QUESTIONS INBLUEBOOKS. SHOW YOUR WORK

1. Let R be the region bounded by the curves $y = x^3$ and $y = x$

A. Sketch R and find its area (Be sure to find ALL points where the curves cross.)

B. Find the volume of the solid generated by rotating R about the x - axis

C. Find the volume of the solid generted when R is rotated about the line x = - 2

2. Find dy/dx

$A.\quad y = (\ln(x))^x \qquad C.\quad y = \ln\left(e^x + e^{-x}\right)$

$C.\quad y = \arcsin(x) + \arctan\left(\sqrt{x}\right)$

3. Evaluate the integrals. If one does not converge show why

A. $\int e^{2x} \cos(3x)\, dx$ B. $\int \frac{4x^2+3}{x^3+x^2+x}\, dx$

C. $\int_0^\infty xe^{-x}\, dx$ D. $\int \frac{\sqrt{x^2-25}}{x}\, dx$ E. $\int \tan^4(x)\, dx$

4. Let $f(x) = x^3 + 3x$

a. show f has an inverse function b. find $(f^{-1})'(4)$

5. Use the integral for arc lenth to find the arc length of the upper semi - circle $y = \sqrt{4 - x^2}$

6. A. Find the general solution of the differential equation

$dy/dx = xe^{x+y}$

B. Find the solution with initial condition $y(0) = 1$

7. Determine if each of the following series are absolutely convergent, conditional convergent, or divrgent. State which test you use in each case and show briefly why it applies

A. $\sum_{n=2}^{\infty} \frac{1}{n\,(\ln(n))^2}$ $B.\ \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ $C.\ \sum_{n=1}^{\infty} \frac{2}{1+n}$

$D.\ \sum_{n=1}^{\infty} \frac{(-1)^n\, 2^{n+1}}{1+3^{n-1}}$ $E.\ \sum_{n=1}^{\infty} \frac{(n!)^2}{(3n)!}$

8. Find the radius of convergence and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{4^n (x+1)^n}{n3^{n+1}}$$

9. A. Compute the Taylor Series for the function f(x) = sin(x) expanding at a = $\pi/2$. (Hint : compute the first few coefficients, and use the pattern you find to write down the k - th coefficient in general).

B. Use Taylor' s Formula to show this series converges to sin(x) for all real numbers

C. write down the 4th Taylor Polynomial $T_4(x)$ and use it to estimate $\sin\left(\pi/2 + .1\right)$

D. find an estimate for the absolute value of the error in part C, $\left| R_4\left(\pi/2 + .1\right) \right|$