QUEENS COLLEGE
DEPARTMENT OF MATHEMATICS

Final Examination
2½ Hours

Mathematics 152 **Spring 2008**

Instructions: **Answer all questions.** **Show all work.**

1. Let R be the region in the plane bounded by the curve $y = 4x^2$ and the lines $y = 12 - 8x$ and $y = 0$.

   a) Sketch region R and determine its area.

   b) Set up, but do not evaluate, the definite integral representing each of the following:

      (i) The volume of the solid generated by rotating R about the line $x = 2$ using the disk-washer method.

      (ii) The volume of the solid generated by rotating R about the line $y = -3$ using the method of cylindrical shells.

2. Find y'.

   a) $y = \left(\tan^{-1}\left(e^x\right)\right)^{x^3}$

   b) $y = 5^{\sin x} + \sin^{-1}(5x)$

3. Evaluate each of the following integrals. If the improper integral diverges, state that as your answer.

   a) $\int \cos^2 x \tan^3 x \, dx$

   b) $\int \frac{x^3}{\sqrt{x^2 - 9}} \, dx$

   c) $\int \frac{1 - 2x - 2x^2}{x^2\left(x^2 + 1\right)} \, dx$

   d) $\int_2^4 \frac{dx}{\sqrt{16 - x^2}}$

   e) $\int \frac{\ln x}{x^4} \, dx$

4. Let $f(x) = 4 + x^3 + \tan\left(\frac{\pi x}{2}\right)$ on the interval $(-1, 1)$.

   a) Use a derivative to show that $f(x)$ is one-to-one and thus has an inverse function.

   b) Given that $(0, 4)$ is a point on the graph of f, compute $g'(4)$, where g is the inverse of f.

5. a) Find $\lim_{x \to 0^+} (1 + \sin 3x)^{\frac{2}{x}}$.

   b) Find the exact value for the arc length of the curve $y = \frac{1}{3}x^{3/2} - x^{1/2}$ from $x = 1$ to $x = 9$.

6. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After one hour there are 420 cells.

a) Determine the number of cells after 2 hours. (to the nearest whole number)

b) When will the population reach 10,000 cells? (to the nearest hundredth)

7. Find the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n7^n}$

8. Suppose $f(x)=x^{-2}$.

a) Find $T_3(x)$, the third Taylor polynomial, centered at $a=1$.

b) Use Taylor's Formula to estimate the largest possible error that can result by approximating $f(x)$ by $T_3(x)$ when x is in the interval $.9 \le x \le 1.1$.

c) Use $T_3(x)$ (and your calculator) to approximate $\frac{1}{(1.1)^2}$ to three decimal places.

9.a) Beginning with the Maclaurin series for $\cos x$, write the Maclaurin series for $\cos(x^2)$.

b) Find a series representation for

(i) $\int \cos(x^2)\,dx$

(ii) $\int_0^1 \cos(x^2)\,dx$

c) Use the fewest number of terms of the series of $\int_0^1 \cos(x^2)\,dx$ to evaluate the definite integral so that the error will be less than .00001.

d) Use your calculator to find the value of $\int_0^1 \cos(x^2)\,dx$ to 6 decimal places.

10. Determine the convergence or divergence of the following series. In the case of an alternating series, determine if the convergence is absolute or conditional. In each case, state what test was used.

a) $\sum_{n=1}^{\infty} \frac{2^n n^n}{5^n n!}$

b) $\sum_{n=1}^{\infty} \frac{5n+1}{n3^n}$

c) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

d) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$

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