Number Patterns Homework

A *sequence* is an ordered list, like $S = (10,\ 2,\ 5,\ 5,\ -2,\ \ldots)$. Each part of a list is called an *entry*. So you would say 10 is the first entry of S, 2 is the second entry of S, and so on. A general sequence S might be written like this:

$$S = (s_1, s_2, s_3, s_4, s_5, \ldots).$$

In other words, the first entry is named s_1, the 100th entry is named s_{100}, and if we're talking about a generic entry of the list, we might write down s_n to represent the n-th entry of the list (where we're not saying what number n is).

Some sequences are nicer than others because it's possible to say what the rule is for each entry. We'll focus on two different ways in which you can describe a rule.

First, you can create a rule that tells you what to do no matter what n you are given. For example, in class we discussed the simple sequence $S = (2, 4, 6, 8, 10, \ldots)$. This was a simple rule because when we notice that $s_1 = 2$, $s_2 = 4$, and $s_3 = 6$, it was not so hard to guess that the n-th entry is always going to be two times n. If we want to write this mathematically, we would write this as $s_n = 2n$ for all positive integers n. We would say that we have found a formula for the n-th entry of the sequence *as a function of* n.

Second, you can create a rule that tells you how to get s_{n+1} if you know what s_n is. This is called a *recurrence relation*. For example, if your rule is that s_{n+1} is always one more than s_n, then your sequence might look like

$$S = (1, 2, 3, 4, 5, 6, \ldots).$$

But your sequence also might look like

$$S = (3, 4, 5, 6, 7, 8, \ldots).$$

Be careful that when you are working with a recurrence relation, you will always have to specify what the first (or first few) numbers of the sequence are.

Homework

Your goal is to come up with three sequences that your groupmates will try to guess.

1. Create a sequence S where s_n is a function of n that is simple to guess.
2. Create a sequence S where s_n is a function of n that is harder to guess.
3. Create a sequence S that is defined by a recurrence relation.

Bring your three sequences to class. We'll spend time guessing them and thinking about their similarities and differences.

If you are interested in learning more about sequences, check out the Online Encyclopedia of Integer Sequences at `http://oeis.org`