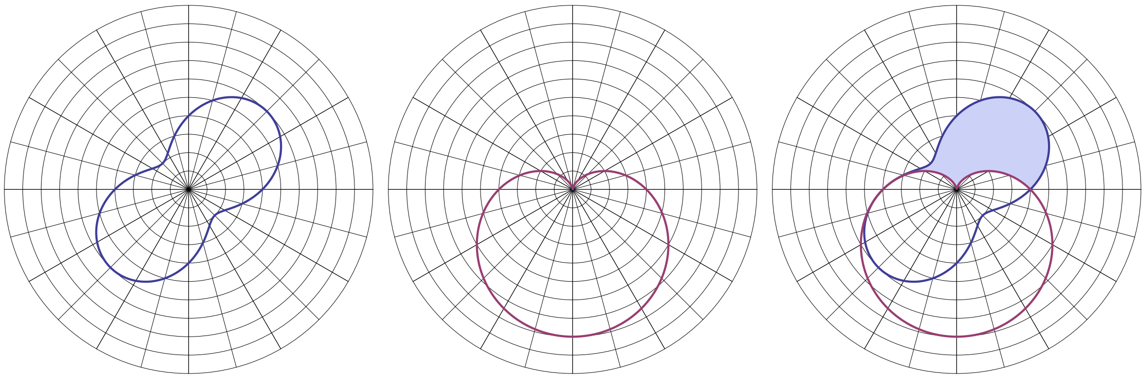
POLAR INTEGRATION PRACTICE – SOLUTIONS

Work through the given steps that are necessary to solve this question.

> Calculate the area of the region **inside** the curve $r = 2 + \sin(2\theta)$ and **outside** the curve $r = 2 - 2\sin(\theta)$.

(1) Draw a rough picture of the two functions on the same set of axes.



(2) Determine which function is further from the origin and which function is closer to the origin in the desired region.

(3) Find the limits of integration.

The curve $r = 2 + \sin(2\theta)$ is further from the origin than the curve $r = 2 - 2\sin(\theta)$ for values of θ from $0 \leq \theta \leq \pi$.

(4) Set up the integral(s) to be completed.

$$\text{Area} = \int_{\theta=0}^{\theta=\pi} \frac{1}{2}\big(2+\sin(2\theta)\big)^2\,d\theta - \int_{\theta=0}^{\theta=\pi} \frac{1}{2}\big(2-2\sin(\theta)\big)^2\,d\theta$$

(5) Do the integration.

$$\begin{aligned}
\text{Area} &= \int_{\theta=0}^{\theta=\pi} \frac{1}{2}\big(2+\sin(2\theta)\big)^2\,d\theta - \int_{\theta=0}^{\theta=\pi} \frac{1}{2}(2)^2\big(1-\sin(\theta)\big)^2\,d\theta && \text{Take out factor of 2.}\\
&= \int_{\theta=0}^{\theta=\pi} \frac{1}{2}\big(4+4\sin(2\theta)+\sin(2\theta)^2\big)\,d\theta - \int_{\theta=0}^{\theta=\pi} 2\big(1-2\sin(\theta)+\sin(\theta)^2\big)\,d\theta && \text{Expand the squares.}\\
&= \int_{\theta=0}^{\theta=\pi} \frac{1}{2}\left(4+4\sin(2\theta)+\frac{1-\cos(4\theta)}{2}\right)d\theta - \int_{\theta=0}^{\theta=\pi} 2\left(1-2\sin(\theta)+\frac{1-\cos(2\theta)}{2}\right)d\theta && \text{Apply trig identity.}\\
&= \int_{\theta=0}^{\theta=\pi} 2\frac{1}{4}+2\sin(2\theta)-\frac{1}{4}\cos(4\theta)\,d\theta - \int_{\theta=0}^{\theta=\pi} 3-4\sin(\theta)-\cos(2\theta)\,d\theta && \text{Multiply in.}\\
&= \int_{\theta=0}^{\theta=\pi} -\frac{3}{4}+4\sin(\theta)+2\sin(2\theta)+\cos(2\theta)-\frac{1}{4}\cos(4\theta)\,d\theta && \text{Combine integrals.}\\
&= -\frac{3\theta}{4}-4\cos(\theta)-\cos(2\theta)+\frac{1}{2}\sin(2\theta)-\frac{1}{16}\sin(4\theta)\Big|_{\theta=0}^{\theta=\pi} && \text{Integrate.}\\
&= \frac{-3\pi}{4}-4(-1-1)-(1-1)+(0-0)-(0-0) && \text{Evaluate.}\\
&= 8-\frac{3}{4}\pi && \text{Simplify.}\\
&\approx 5.64381 && \text{Approximation.}
\end{aligned}$$