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It is a parabolic cylinder.

Example. $y^2 + z^2 = 1$. $\longleftarrow$ x is not in this equation.
For any choice of $x = k$, the surface looks like a unit circle.

Quadric surfaces

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Ax^2

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Example. $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1.$

When $z = 0$, $x^2 + \frac{y^2}{9} = 1$

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When $z = k$, $x^2 + \frac{y^2}{9} = 1 - \frac{k^2}{4}$ is an ellipse when $1 - \frac{k^2}{4} \geq 0$.

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When $x = k$, $\frac{y^2}{9} + \frac{z^2}{4} = 1 - k^2$ is an ellipse

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Every slice is an ellipse $\rightsquigarrow$ surface is an ellipsoid.

Example. $z = y^2 - x^2$

Slices	$x = k$	$y = k$	$z = k$
Eqn Format	$z = y^2 - k^2$	$z = k^2 - x^2$	$k = y^2 - x^2$
Conic section			
Sketches			

Assemble together:

Need to know

- There are six different families of quadric surfaces.

Ellipsoid (Sphere) $+\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Cone $+\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$
Elliptic paraboloid $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	Hyperboloid of one sheet $+\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Hyperbolic paraboloid $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	Hyperboloid of two sheets $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Online Resources:

https://www.youtube.com/watch?v=LBiiOEiD3Yk

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- Matching equations to surfaces.

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Ellipsoid (Sphere)	Cone
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Elliptic paraboloid	**Hyperboloid of one sheet**
$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	$+\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
Hyperbolic paraboloid	**Hyperboloid of two sheets**
$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

- Matching equations to surfaces.
- More variety than conic sections but same building blocks.

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- Matching equations to surfaces.
- More variety than conic sections but same building blocks.
- How to find slices, assemble to a rough sketch.

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