

Functions of Several Variables

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 - ▶ A curve represents points in the domain at the same “height”.

The domain of a function of two variables

Example. What is the domain of the functions

$$f(x, y) = \frac{\sqrt{x + y + 1}}{x - 1} \quad \text{and} \quad g(x, y) = x \ln(y^2 - x)?$$

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Example. Sketch the following functions

$$f(x, y) = 6 - 3x - 2y \quad \text{and} \quad g(x, y) = \sqrt{9 - x^2 - y^2}.$$

Level curves

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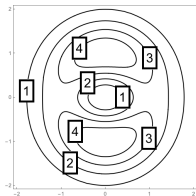
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Example. What surface corresponds to this contour map?



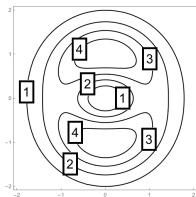
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Example. Sketch the level curves of the function $h(x, y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$.



More variables

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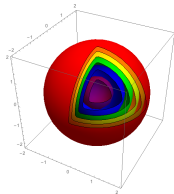
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This gives a **surface** on which the function has a constant value.

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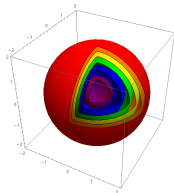
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Think: Which positions in this room have the same temperature?