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This works with more variables too.

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Clairaut's Theorem

Example. Calculate all second-order partial derivatives of

$$f(x, y) = x^3 + x^2y^3 - 2y^2.$$

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Interpretation of partial derivatives

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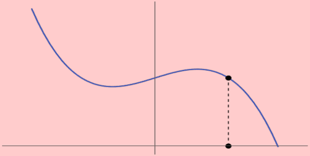

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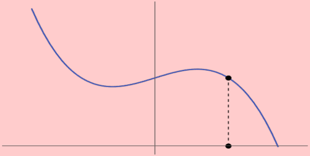

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$$y = f(x)$$

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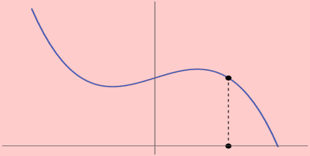

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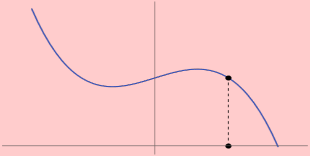

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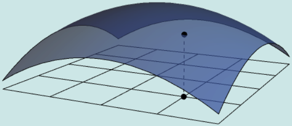

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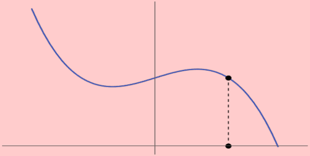

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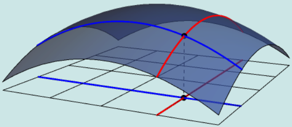

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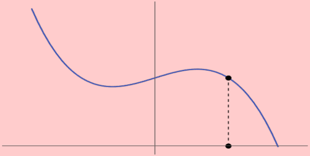

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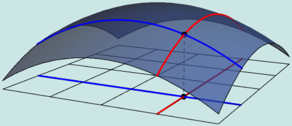

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"If y is fixed, what is the rate of change of $f(x, y)$ as x changes?"

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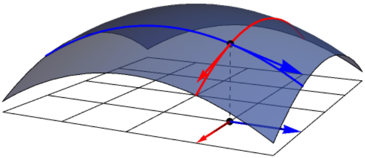
$\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at (x_0, y_0) are slopes of tangent lines along the surface.



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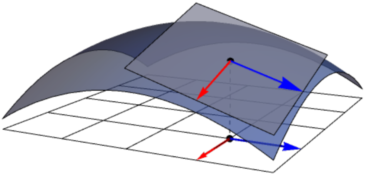

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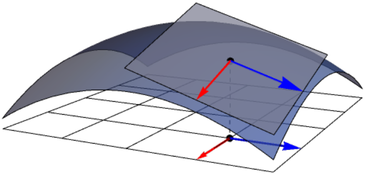

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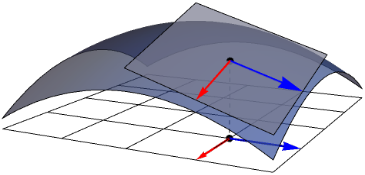

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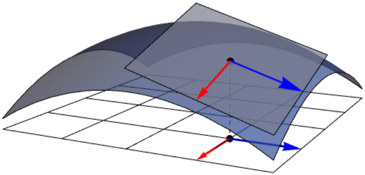

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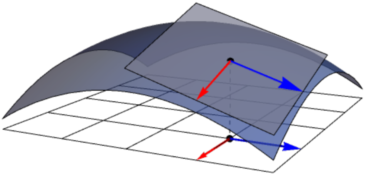

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Theorem. If the partial derivatives f_x and f_y **exist** near (a, b) and **are continuous** at (a, b), then f is differentiable at (a, b).

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The equation of this tangent plane is easy. (point-slope)

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A linear approximation near $\vec{\mathbf{v}}_0 = (w_0, x_0, y_0, z_0)$ would be

$$f(\vec{\mathbf{v}}) - f(\vec{\mathbf{v}}_0) \approx f_w(\vec{\mathbf{v}}_0)(w - w_0) + f_x(\vec{\mathbf{v}}_0)(x - x_0) + f_y(\vec{\mathbf{v}}_0)(y - y_0) + f_z(\vec{\mathbf{v}}_0)(z - z_0)$$

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$$dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy$$

dz is a *approximation* for how much z actually changes.

Example. If $z = x^2 + 3xy - y^2$, find dz.

$dz =$

Conclusion: If x changes from $2 \to 2.05$, $dx =$ ______

If y changes from $3 \to 2.96$, $dy =$ ______.

We would expect z to change by ____________.

The true change is .6449.