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(r, θ) are polar coords of xy-plane
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and z is the "distance" to xy-plane

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Useful when problems involve *symmetry about an axis*. Cylinder, Paraboloids, Cones w/flat bases

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Useful when problems involve *symmetry about a point*
Spheres, Cones with curved bases.

Converting from cartesian coordinates

Cylindrical coordinates

Conversion:

$x = r\cos\theta \quad y = r\sin\theta \quad z = z$

$r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x} \quad z = z$

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Spherical coordinates

Conversion: $x = \rho\sin\phi\cos\theta$

$z = \rho\cos\phi \quad y = \rho\sin\phi\,\sin\theta$

$\rho^2 = x^2 + y^2 + z^2; \quad \tan\phi = \frac{\sqrt{x^2+y^2}}{z}$

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Practice

Cylindrical coordinates

Practice changing coordinates:

$(r,\theta,z)=(2,\frac{2\pi}{3},1)$; $(x,y,z)=(3,-3,7)$

Identify cyl. coord. equations:

2. $r=2$; $z=r^2$; $r^2-2z^2=4$

3. Sketch $r^2 \leq z \leq 2-r^2$

Convert to cylindrical coordinates

4. $x^2+y^2+z^2=2$; $x^2+y^2=2y$

5. Give solid between $x^2+y^2=1$ and $x^2+y^2+z^2=4$.

6. $\left\{\begin{array}{c} -2 \leq x \leq 2 \\ -\sqrt{4-x^2} \leq y \leq \sqrt{4-x^2} \\ \sqrt{x^2+y^2} \leq z \leq 2 \end{array}\right\}$

Spherical coordinates

Practice changing coordinates:

$(\rho,\theta,\phi)=(2,\frac{\pi}{4},\frac{\pi}{3})$;$(x,y,z)=(-1,1,\sqrt{6})$

Identify sph. coord. equations:

2. $\phi=\frac{\pi}{3}$; $\rho\sin\phi=2$; $\rho=2\cos\phi$

3. Sketch $\left(2 \leq \rho \leq 3 \ \& \ \frac{\pi}{2} \leq \phi \leq \pi\right)$
Sketch $\left(0 \leq \phi \leq \frac{\pi}{3} \ \& \ \rho \leq 2\right)$

Convert to spherical coordinates

4. $z=x^2+y^2$; $z=x^2-y^2$

5. Give solid inside $x^2+y^2+z^2=4$, above xy-plane, below $z=\sqrt{x^2+y^2}$.

6. $\left\{\begin{array}{c} 0 \leq x \leq 1 \\ 0 \leq y \leq \sqrt{1-x^2} \\ \sqrt{x^2+y^2} \leq z \leq \sqrt{2-x^2-y^2} \end{array}\right\}$

Course Evaluation

Please comment on:

1. Prof. Chris's effectiveness as a teacher.
2. Prof. Chris's contribution to your learning.
3. The course material: What you enjoyed and/or found challenging.
4. Is there anything you would change about the course?
5. The lecture portion of the class included electronic slides.
   In what ways did this enhance or detract from your learning?
6. The assigned Webwork and homework assignments.
7. Is there anything else Prof. Chris should know?

Place completed evaluations in the provided folder.

I will be in my office, Kissena Hall, Room 355.