Course Notes

Multivariable Calculus, Fall 2014

Queens College, Math 201

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http://qcpages.qc.edu/~chanusa/courses/201/14f/

Class Introductions

Arrange yourselves into groups of four or five people,
With people you **don't know**.

- Introduce yourself. (your name, where you're from, your major)
- What brought you to this class?
- Fill out **the blank side of** your notecard:
  - Write your name. (Stylize if you wish.)
  - Write a few words related to your name.
  - *Draw* something in the remaining space.
- Discuss with your groupmates why you wrote what you wrote.
- Exchange contact information. (phone / email / other)
- **Discuss!** What is *Calculus*?
  - Brainstorm! How do you convey *Calculus* to friends?
  - Organize into themes.
- How do these ideas translate to *multivariable* calculus?

To do well in this class:

- **Form good study groups.**
  - Discuss homework and classwork. Study for exams.
  - Bounce around ideas, topics, questions.
  - You will depend on this group.
- **Put in the time.**
  - Four credits = (at least) twelve hours / week out of class.
  - Homework stresses key concepts from class; learning takes time.
- **Come to class prepared.**
  - Review previous day's sections, notes.
  - Do the homework & prepare to present.
  - Preview the new day's sections. Download notes!
- **Stay in contact.**
  - If you are confused, ask questions (in class and out).
  - Don't fall behind in coursework or homework.
  - I need to understand your concerns.

All homeworks posted online; first one (many parts) due Tuesday.

Homework policy:

There are two types of homework in this class:

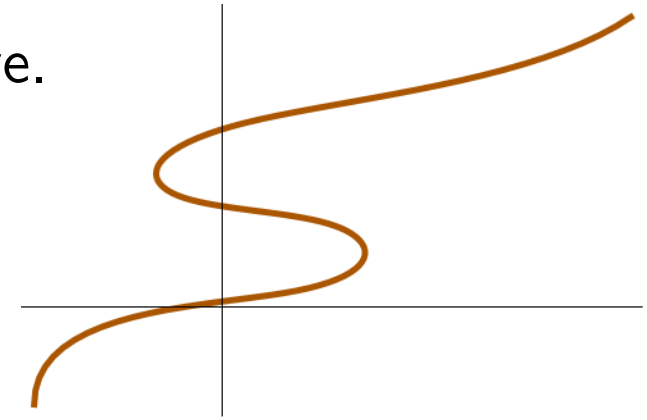
- **Daily:** Written / Presentation Homework.
  - A list of questions from the textbook to practice.
  - If a question is hard, you should practice **more** like it.
  - Presentations at beginning of the next class.
    - Write up solution in bullet-point format.
    - Present the solution to the class & answer questions.
  - One of only two bonus point opportunities in this class.
  - Starts Tuesday September 2! (+ Blackboard quiz)
- **Weekly:** Online Homework.
  - Using online homework called Webwork.
    - Link on webpage to:
      `http://192.195.176.176/webwork2/QC201/`
    - Your username: QC email username.
    - Initial password: CUNYFirst ID #
  - First assignment due Thursday September 4. (13 Qs)
  - $*$ Get started early! $*$

`http://qcpages.qc.edu/~chanusa/courses/201/14f/forum.html`

Parametric Curves

Imagine a particle traveling along this curve.

Is the curve a function? ($y = f(x)$?)



As an alternative, we can write
the x-coordinate and the y-coordinate
of the particle as a function of "time".

(Write $x = f(t)$ and $y = g(t)$.)

This pair of functions is called the **parametric equations** of the curve.

And the variable t is called a **parameter**.

Note: The domain of t is often $(-\infty, \infty)$ or an interval $a \leq t \leq b$.

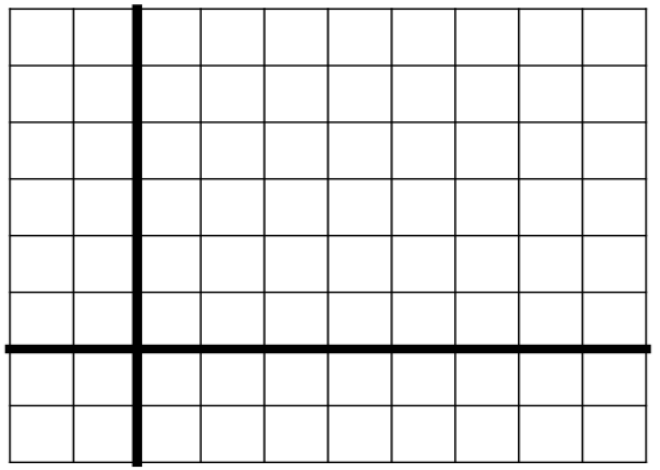
Goal 1: *Understand* parametric curves. (Today)

Goal 2: *Do calculus* using parametric curves. (Next time)

Sketching Parametric Curves

What is the shape of a curve given by parametric equations?

- By hand ⟵ How do you plot $y = f(x)$?
- Use a calculator or computer



Example. Plot the curve defined by $x(t) = t^2 - 2t$ and $y(t) = t + 1$.

t	-1	0	1	2	3	4
$x(t)$						
$y(t)$						

The shape of the curve is _____.

Should we have known this?

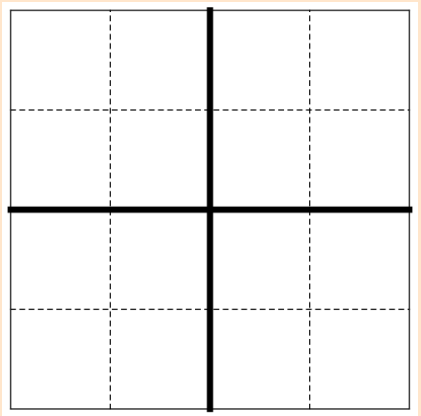
Key concept: Eliminate the parameter t to combine $x = f(t)$ and $y = g(t)$ into a "normal" function $y = F(x)$ or $x = F(y)$.

Solve for t in second equation: $t = y - 1$ and plug in:
$x = (y-1)^2 - 2(y-1) = y^2 - 4y + 3$, a "sideways parabola".

Around and Around

Example. Plot the curve defined by $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.

Plot points or solve directly.

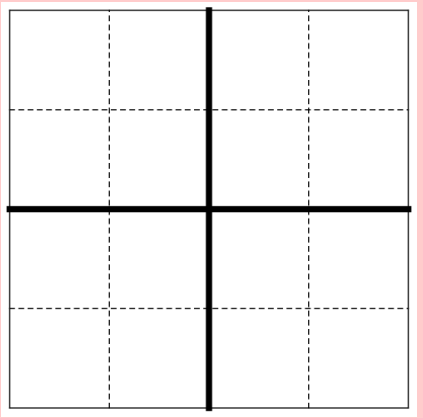


$y = \sin(\cos^{-1} x)$
$y = \sqrt{1 - x^2}$
$y^2 = 1 - x^2$
$x^2 + y^2 = 1$

A circle! But you knew that.
$\cos^2 t + \sin^2 t = 1$

Starts at $t = 0$: $(1, 0)$ and goes around counterclockwise.

Example. Is this the same as $x = \sin 2t$, $y = \sin 2t$, $0 \le t \le 2\pi$?



Question: What is $x^2 + y^2$?

The figures traced out (the curves) are the same but the functions **are not** the same.

You need to know your trig functions and values at certain points!!!!

Circumnavigation

If we want to draw a circle at some other place

$$(x-h)^2+(y-k)^2=r^2,$$

set $x-h=r\cos t$ and $y-k=r\sin t$.

In other words, use the parametric equations

$$x(t)=r\cos t+h \quad \text{and} \quad y(t)=r\sin t+k.$$

Try it out! Get out your graphing calculator TI-(≤ 86).

Switch to Parametric mode: `MODE` $\downarrow\downarrow\downarrow$ `PAR` (`Enter`).

Set the domain of T to be from 0 to 2π.
`WINDOW: Tmin = 0`, `Tmax = 2π`, `Tstep = π/10`.

Enter the equations X_1 `= 3 cos(T) + 2` and Y_1 `= 3 sin(T) + 4`.

This plots a circle of radius 3 centered at $(2,4)$.

Computers to the rescue

Calculators and computers can graph much more complicated curves.

$$x_1(t) = t + 2\sin(2t) \quad \text{and} \quad y_1(t) = t + 2\cos(5t)$$

$$x_2(t) = 1.5\cos t - \cos 30t \quad \text{and} \quad y_2(t) = 1.5\sin t - \sin 30t$$

$$x_3(t) = \sin(t + \cos 100t) \quad \text{and} \quad y_3(t) = \cos(t + \sin 100t)$$

Tools:

- Wolfram Alpha `http://www.wolframalpha.com/`
- More powerful is Wolfram *Mathematica*. Get license from MyQC: `myqc.qc.cuny.edu/Academics/mathematics/Pages3/access.aspx`
- Online plotter: `desmos.com` Put $(f(t), g(t))$ in parentheses. `https://www.desmos.com/calculator/ndgy5rppqh`

Next time: What is the shape of a parametric curve? What is the length of a parametric curve? What about polar coordinates?

Before then: Work on homework to present in class Wednesday. Email me contact info, do syllabus quiz. Play with parametric eqns.