Finding slope on a parametric curve

When y is a function of x, what is the slope of the tangent line?

For a parametric curve $\{x = f(t),\ y = g(t)\}$,

Think of y as a function of x. Then $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$, so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ if _____.

- Curve has a horizontal tangent where $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$.
- Curve has a vertical tangent where $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$.
- *Question:* What is true when $\frac{dx}{dt} = 0$ AND $\frac{dy}{dt} = 0$?

We can use the chain rule again to find $\frac{d^2y}{dx^2}$, but be careful!

$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) =$ __________. ($\frac{dy}{dx}$ is a function of _____.)

Important: $\frac{d^2y}{dx^2} \neq \frac{d^2y}{dt} \Big/ \frac{d^2x}{dt}$!!!!!

Slope of tangent line

Example. What is the tangent line to the curve $\begin{cases} x(t) = t^2 \\ y(t) = t^3 - 3t \end{cases}$ at (3,0)?

Question: What is t there?

Question: What is the slope there?

Question: So what is the tangent line there?

Sketching the curve

$$\begin{cases} x(t) = t^2 \\ y(t) = t^3 - 3t \end{cases}$$

Let's now sketch the curve.

Question: Where are there horizontal and vertical tangents?

- Horizontal:
- Vertical: (Must check?)

Question: Where is the curve concave up? concave down?

- Calculate
$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{3t^2-3}{2t}\right)}{\frac{dx}{dt}} = \frac{\frac{3}{2} + \frac{3}{2t^2}}{2t} = \frac{\frac{3}{2}}{2}\frac{1}{t}\left(1 + \frac{1}{t^2}\right) = \frac{3(t^2+1)}{4t^3}.$$

Put it all together:

t	x	y
-3		
-1		
0		
1		
3		

Polar coordinates

Polar coordinates are an alternate way to think about points in 2D.

Conversions:

$$\begin{array}{c} x = r\cos\theta \\ y = r\sin\theta \end{array} \quad \longleftrightarrow \quad \begin{array}{c} r^2 = x^2 + y^2 \\ \tan\theta = \frac{y}{x} \end{array}$$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin					
cos					
tan					

Need to know

Changing coordinates:

$(r,\theta)=(2,-\frac{2\pi}{3})$ then $(x,y)=$

$(x,y)=(-1,1)$ then $(r,\theta)=$

Identifying polar equations:

$\theta = 1$ $r = 2$

$r = 2\cos\theta$

$r = \cos 2\theta$ $r = 1 + \sin\theta$

Using your calculator:

Switch to Polar mode:

`MODE` $\downarrow\downarrow\downarrow$ `POL` (`Enter`).

Also: `desmos.com` or Mathematica

Tangents to polar curves

Given a polar curve $r = f(\theta)$, we want to know $\frac{dy}{dx}$.
Just as before, think of y as a function of x. Then $\frac{dy}{d\theta} = \frac{dy}{dx} \cdot \frac{dx}{d\theta}$,

We conclude: $$\frac{dy}{dx} = \frac{\frac{d}{d\theta}y}{\frac{d}{d\theta}x} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{\left(r\cos\theta + \frac{dr}{d\theta}\sin\theta\right)}{\left(r\sin\theta + \frac{dr}{d\theta}\cos\theta\right)} \text{ if } \_\_\_\_\_ .$$

Example. Find the slope of the tangent line to the curve $r = 2\sin\theta$ at cartesian coordinates $(x, y) = (2, 0)$.