# Coordinates, Now with More Dimensions....

## 2D Coordinates

Variables: $x$ (indep), $y$ (dep)

Axes: $x$-axis $\perp$ $y$-axis

## 3D Coordinates

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Coordinates of a point: $(a, b)$
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# Continued Comparison

**2D Equations**

**Lines:**

$x = 2$ defines

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$x = 2$ is a plane parallel to the $yz$-plane.
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All points distance $r$ from $(h, k, \ell)$:

$$x^2 + y^2 + z^2 = r^2$$

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# Vectors

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Distributive laws:
$(a + b) \cdot \vec{\mathbf{v}} = a \cdot \vec{\mathbf{v}} + b \cdot \vec{\mathbf{v}}$
$c \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}}) = c \cdot \vec{\mathbf{u}} + c \cdot \vec{\mathbf{v}}$

# Grasping the magnitude of the situation

To write a vector in coordinates, place the tail at the origin and find the coordinates of the head.

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- $\vec{\mathbf{u}} = \langle 1, 4, 3 \rangle$, then $|\vec{\mathbf{u}}| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26}$.
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- Find the length of $\vec{\mathbf{u}}$ and divide by it!

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Three special vectors: $\vec{\mathbf{i}} = \langle 1, 0, 0 \rangle$, $\vec{\mathbf{j}} = \langle 0, 1, 0 \rangle$, $\vec{\mathbf{k}} = \langle 0, 0, 1 \rangle$.

- Alternate form: $\vec{\mathbf{u}} = \vec{\mathbf{i}} + 4\vec{\mathbf{j}} + 3\vec{\mathbf{k}}$. **standard basis vectors**

We add vectors componentwise. And multiply scalars componentwise.

- $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle 1 + 0, 4 + (-1), 3 + (-3) \rangle = \langle 1, 3, 0 \rangle$
- $\pi\vec{\mathbf{u}} = \pi\langle 1, 4, 3 \rangle = \langle \pi, 4\pi, 3\pi \rangle$ Stretch it!

**Magnitude** is a synonym for **length**, written $|\vec{\mathbf{v}}|$ or $\|\vec{\mathbf{v}}\|$.
You may need to find the **unit vector** in the same direction as $\vec{\mathbf{u}}$.

- Find the length of $\vec{\mathbf{u}}$ and divide by it!
- Example. Unit vector of $\langle 2, -1, -2 \rangle$ is

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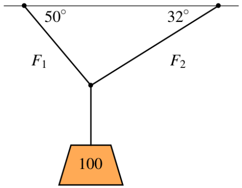


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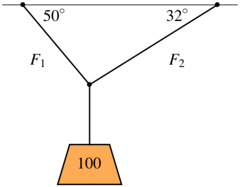


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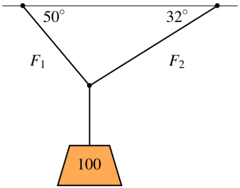


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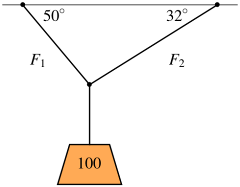


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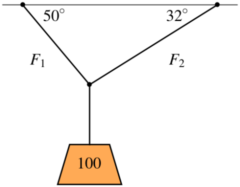


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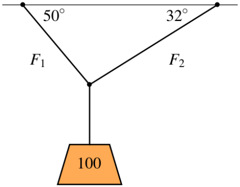


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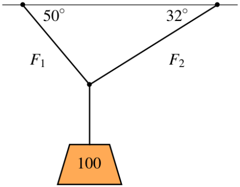


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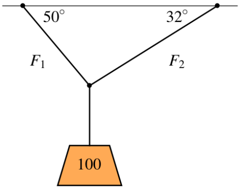


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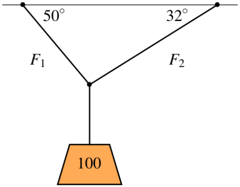


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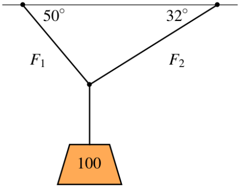


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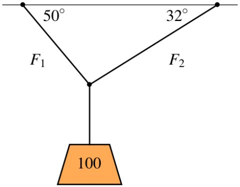


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$-\cos 50\, F_1 + \cos 32\, F_2 = 0$

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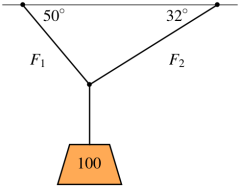


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$-\cos 50\, F_1 + \cos 32\, F_2 = 0$ and $\sin 50\, F_1 + \sin 32\, F_2 - 100 = 0$

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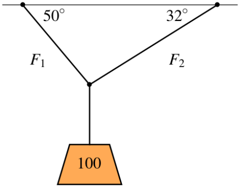


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- Use equilibrium to get a system of equations, solve.

$-\cos 50\, F_1 + \cos 32\, F_2 = 0$ and $\sin 50\, F_1 + \sin 32\, F_2 - 100 = 0$

Solving gives $F_1 \approx 85$ lb and $F_2 \approx 65$ lb.