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In our example, $s = \sqrt{2}t$, so $t = \frac{s}{\sqrt{2}}$. Substituting,

$$\vec{\mathbf{r}}(s) = \cos \frac{s}{\sqrt{2}}\,\vec{\mathbf{i}} + \sin \frac{s}{\sqrt{2}}\,\vec{\mathbf{j}} + \frac{s}{\sqrt{2}}\,\vec{\mathbf{k}}.$$

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The circle that lies along the curve has radius $1/\kappa$. (**!**)

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Question: Should $\kappa(t)$ be a constant?

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- a_N Toward $\vec{\mathbf{N}}$: $a_N = \kappa v^2$. Curvature times speed squared!

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Key idea: Understand $\vec{\mathbf{a}}$ in terms of the Frenet frame:
How much of the acceleration is toward $\vec{\mathbf{T}}$, $\vec{\mathbf{N}}$, and $\vec{\mathbf{B}}$?

Differentiate $\vec{\mathbf{v}}(t) = v(t)\vec{\mathbf{T}}(t)$. (magnitude (speed) times unit direction)

$$\vec{\mathbf{a}} = v'\vec{\mathbf{T}} + v\vec{\mathbf{T}}' = v'\vec{\mathbf{T}} + v(|\vec{\mathbf{T}}'|\vec{\mathbf{N}}) = v'\vec{\mathbf{T}} + \kappa v^2\vec{\mathbf{N}}.$$

- All acceleration is toward $\vec{\mathbf{T}}$ and $\vec{\mathbf{N}}$. (Not to $\vec{\mathbf{B}}$.)

a_T Toward $\vec{\mathbf{T}}$: $a_T = v'$ is rate of change of speed.

a_N Toward $\vec{\mathbf{N}}$: $a_N = \kappa v^2$. Curvature times speed squared!

Solve for a_T, a_N in terms of $\vec{\mathbf{r}}(t)$.

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So $a_T = v' = \frac{\vec{\mathbf{v}} \cdot \vec{\mathbf{a}}}{v}$

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So $a_T = v' = \frac{\vec{\mathbf{v}} \cdot \vec{\mathbf{a}}}{v} = \frac{\vec{\mathbf{r}}'(t) \cdot \vec{\mathbf{r}}''(t)}{|\vec{\mathbf{r}}'(t)|}$

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Nice symmetry!

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Example. Find tang'l, normal comp's of acceleration for $\vec{\mathbf{r}} = \langle t, 2t, t^2\rangle$.