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This works with more variables too.

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Clairaut's Theorem

Example. Calculate all second-order partial derivatives of

$$f(x, y) = x^3 + x^2y^3 - 2y^2.$$

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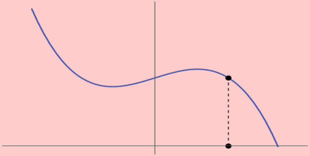
$$f_{xyzz} = f_{zxyz} = f_{zyzx} = \cdots$$

Interpretation of partial derivatives

Function of one variable



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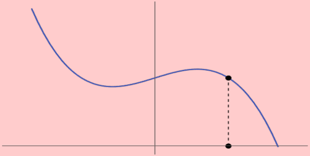

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$$y = f(x)$$

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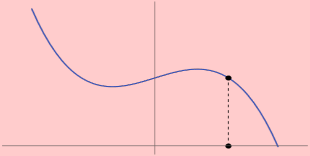

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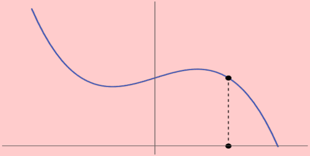

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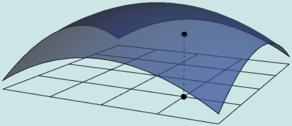

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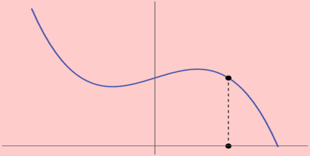

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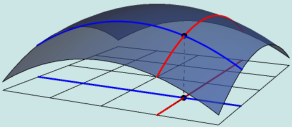

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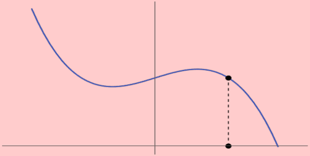

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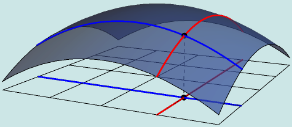

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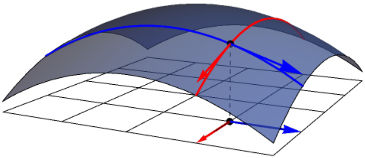
"If y is fixed, what is the rate of change of $f(x, y)$ as x changes?"

Tangenty

$\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at (x_0, y_0) are slopes of tangent lines along the surface.



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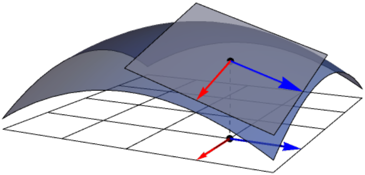
They lie in the **tangent plane**.



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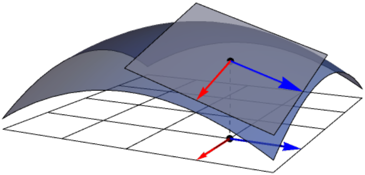

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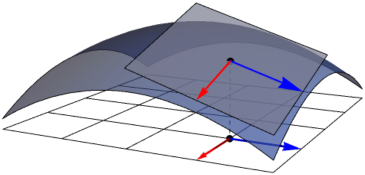

For any curve on the surface through $\left(x_0, y_0, f(x_0, y_0)\right)$, the tangent line to the curve would be in this plane too.

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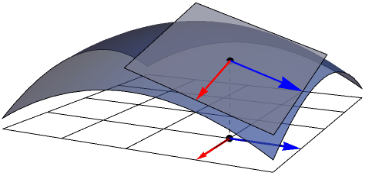

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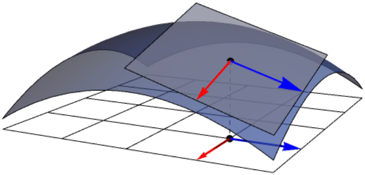

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Key Idea: When the tangent plane to the surface is a good approximation for the the graph when (x, y) is near (a, b), we say that f is **differentiable** at (a, b).

Theorem. If the partial derivatives f_x and f_y **exist** near (a, b) and **are continuous** at (a, b), then f is differentiable at (a, b).

Planey

The equation of this tangent plane is easy. (point-slope)

$$(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

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A linear approximation near $\vec{\mathbf{v}}_0 = (w_0, x_0, y_0, z_0)$ would be

$$f(\vec{\mathbf{v}}) - f(\vec{\mathbf{v}}_0) \approx f_w(\vec{\mathbf{v}}_0)(w - w_0) + f_x(\vec{\mathbf{v}}_0)(x - x_0) + f_y(\vec{\mathbf{v}}_0)(y - y_0) + f_z(\vec{\mathbf{v}}_0)(z - z_0)$$

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Differentials

Differentials are the other way to understand linear approximations. How much does z change as x and y change?

$$dz = \frac{\partial z}{\partial x}\, dx + \frac{\partial z}{\partial y}\, dy$$

dz is a *approximation* for how much z actually changes.

Example. If $z = x^2 + 3xy - y^2$, find dz.
$dz =$

Conclusion: If x changes from $2 \to 2.05$, $dx =$ ______
If y changes from $3 \to 2.96$, $dy =$ ______.
We would expect z to change by ____________.

The true change is .6449.