Generalizations of Polar Coordinates

When 2-dim'l regions D have radial flavors, we use polar coordinates.

When 3-dim'l regions E have radial flavors, there are two choices:

Generalizations of Polar Coordinates

When 2-dim'l regions D have radial flavors, we use polar coordinates.

When 3-dim'l regions E have radial flavors, there are two choices:

Cylindrical coordinates

Spherical coordinates

Generalizations of Polar Coordinates

When 2-dim'l regions D have radial flavors, we use polar coordinates.

When 3-dim'l regions E have radial flavors, there are two choices:

Cylindrical coordinates

A point can have coords (r, θ, z):

(r, θ) are polar coords of xy-plane
(r is the distance from the z-axis)
and z is the "distance" to xy-plane

Spherical coordinates

Generalizations of Polar Coordinates

When 2-dim'l regions D have radial flavors, we use polar coordinates.

When 3-dim'l regions E have radial flavors, there are two choices:

Cylindrical coordinates

A point can have coords (r, θ, z):

(r, θ) are polar coords of xy-plane (r is the distance from the z-axis) and z is the "distance" to xy-plane

Useful when problems involve *symmetry about an axis*. Cylinder, Paraboloids, Cones w/flat bases

Spherical coordinates

Generalizations of Polar Coordinates

When 2-dim'l regions D have radial flavors, we use polar coordinates.

When 3-dim'l regions E have radial flavors, there are two choices:

Cylindrical coordinates

A point can have coords (r, θ, z):

(r, θ) are polar coords of xy-plane (r is the distance from the z-axis) and z is the "distance" to xy-plane

Useful when problems involve *symmetry about an axis*. Cylinder, Paraboloids, Cones w/flat bases

Spherical coordinates

A point can have coords (ρ, θ, ϕ).

ρ is the distance from the origin
θ is same as in polar
ϕ is $\angle$ between $+z$ axis and $\overline{OP}$.

Generalizations of Polar Coordinates

When 2-dim'l regions D have radial flavors, we use polar coordinates.

When 3-dim'l regions E have radial flavors, there are two choices:

Cylindrical coordinates

A point can have coords (r, θ, z):

(r, θ) are polar coords of xy-plane (r is the distance from the z-axis) and z is the "distance" to xy-plane

Useful when problems involve *symmetry about an axis*. Cylinder, Paraboloids, Cones w/flat bases

Spherical coordinates

A point can have coords (ρ, θ, ϕ).

ρ is the distance from the origin
θ is same as in polar
ϕ is $\angle$ between $+z$ axis and $\overline{OP}$.

Useful when problems involve *symmetry about a point*
Spheres, Cones with curved bases.

Converting from cartesian coordinates

Cylindrical coordinates

Conversion:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$
$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$$

Converting from cartesian coordinates

Cylindrical coordinates

Conversion:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$
$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$$

$$dV = r\,dr\,d\theta\,dz$$

Converting from cartesian coordinates

Cylindrical coordinates

Conversion:

$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$

$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$

$$dV = r\,dr\,d\theta\,dz$$

Spherical coordinates

Conversion: $x = \rho\sin\phi\cos\theta$

$z = \rho\cos\phi \qquad y = \rho\sin\phi\,\sin\theta$

$\rho^2 = x^2 + y^2 + z^2; \qquad \tan\phi = \frac{\sqrt{x^2+y^2}}{z}$

Converting from cartesian coordinates

Cylindrical coordinates

Conversion:

$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$

$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$

$$dV = r\,dr\,d\theta\,dz$$

Spherical coordinates

Conversion: $x = \rho\sin\phi\cos\theta$

$z = \rho\cos\phi$ $y = \rho\sin\phi\,\sin\theta$

$\rho^2 = x^2 + y^2 + z^2;$ $\tan\phi = \frac{\sqrt{x^2+y^2}}{z}$

$$dV = \rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

Practice

Cylindrical coordinates

Practice changing coordinates:

$(r,\theta,z)=(2,\frac{2\pi}{3},1)$; $(x,y,z)=(3,-3,7)$

Identify cyl. coord. equations:

2. $r=2$; $z=r^2$; $r^2-2z^2=4$
3. Sketch $r^2\le z\le 2-r^2$

Convert to cylindrical coordinates

4. $x^2+y^2+z^2=2$; $x^2+y^2=2y$
5. Give solid between $x^2+y^2=1$ and $x^2+y^2+z^2=4$.
6. $$\begin{Bmatrix} -2\le x\le 2 \\ -\sqrt{4-x^2}\le y\le \sqrt{4-x^2} \\ \sqrt{x^2+y^2}\le z\le 2 \end{Bmatrix}$$

Spherical coordinates

Practice changing coordinates:

$(\rho,\theta,\phi)=(2,\frac{\pi}{4},\frac{\pi}{3})$;$(x,y,z)=(-1,1,\sqrt{6})$

Identify sph. coord. equations:

2. $\phi=\frac{\pi}{3}$; $\rho\sin\phi=2$; $\rho=2\cos\phi$
3. Sketch $\left(2\le\rho\le 3\ \&\ \frac{\pi}{2}\le\phi\le\pi\right)$
   Sketch $\left(0\le\phi\le\frac{\pi}{3}\ \&\ \rho\le 2\right)$

Convert to spherical coordinates

4. $z=x^2+y^2$; $z=x^2-y^2$
5. Give solid inside $x^2+y^2+z^2=4$, above xy-plane, below $z=\sqrt{x^2+y^2}$.
6. $$\begin{Bmatrix} 0\le x\le 1 \\ 0\le y\le \sqrt{1-x^2} \\ \sqrt{x^2+y^2}\le z\le \sqrt{2-x^2-y^2} \end{Bmatrix}$$

Course Evaluation

Please comment on:

1. Prof. Chris's effectiveness as a teacher.
2. Prof. Chris's contribution to your learning.
3. The course material: What you enjoyed and/or found challenging.
4. Is there anything you would change about the course?
5. The lecture portion of the class included electronic slides.
   In what ways did this enhance or detract from your learning?
6. The assigned Webwork and homework assignments.
7. Is there anything else Prof. Chris should know?

Place completed evaluations in the provided folder.

I will be in my office, Kiely Tower, Room 606.