Course Notes

Multivariable Calculus, Fall 2015

Queens College, Math 201

Prof. Christopher Hanusa

`http://qc.edu/~chanusa/courses/201/15/`

Class Introductions

Arrange yourselves into groups of four or five people,
With people you **don't know**.

- ▶ Introduce yourself. (your name, where you're from, your major)
- ▶ What brought you to this class?
- ▶ Fill out **the blank side of** your notecard:
    - ▶ Write your name. (Stylize if you wish.)
    - ▶ Write some words about how I might remember you & your name.
    - ▶ *Draw* something (anything!) in the remaining space.
- ▶ *Small talk suggestion:* What kept you busy this summer?
- ▶ Exchange contact information. (phone / email / other)

- ▶ **Discuss!** What is *Calculus*?
    - ▶ Brainstorm! How do you convey *Calculus* to friends?
    - ▶ Organize into themes.

- ▶ How do these ideas translate to *multivariable* calculus?

To do well in this class:

▶ **Form good study groups !!!!!!!!**
  - ▶ Discuss classwork and homework. Study for exams.
  - ▶ Bounce around ideas, topics, questions.
  - ▶ You will depend on this group.

▶ **Put in the time.**
  - ▶ Four credits = (at least) twelve hours / week out of class.
  - ▶ Homework stresses key concepts from class; learning takes time.

▶ **Come to class prepared.**
  - ▶ Review previous day's sections, notes.
  - ▶ Do the homework & prepare to present.
  - ▶ Preview the new day's sections. Download notes!

▶ **Stay in contact.**
  - ▶ If you are confused, ask questions (in class and out).
  - ▶ Don't fall behind in coursework or homework.
  - ▶ I need to understand your concerns.

All homeworks posted online; first one (many parts) due Tuesday.

Homework policy:

There are two types of homework in this class:

▶ **Daily:** Written / Presentation Homework.
  - ▶ A list of questions from the textbook to practice.
  - ▶ If a question is hard, you should practice **more** like it.
  - ▶ Presentations at beginning of the next class.
    - ▶ Write up solution in bullet-point format.
    - ▶ Present the solution to the class & answer questions.
  - ▶ One of only two bonus point opportunities in this class.
  - ▶ Starts Tuesday September 1! (+ Blackboard quiz)

▶ **Weekly:** Online Homework.
  - ▶ Using online homework called Webwork.
    - ▶ Link on webpage to:
      `http://192.195.176.176/webwork2/QC201/`
    - ▶ Your username: QC email username. (like Christopher.Hanusa55)
    - ▶ Initial password: CUNYFirst ID # (like 10111213)
  - ▶ First assignment due Thursday September 3. (13 Qs)
  - ▶ ∗ Get started early! ∗

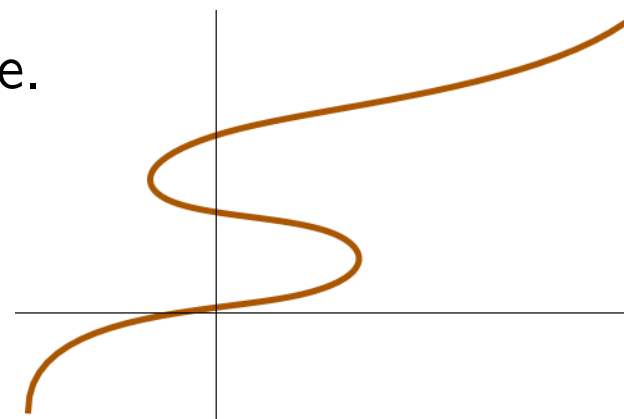`http://qc.edu/~chanusa/courses/201/15/forum.html`

Parametric Curves

Imagine a particle traveling along this curve.

Is the curve a function? $(y = f(x)?)$

As an alternative, we can write
the x-coordinate and the y-coordinate
of the particle as a function of "time".

$$\text{(Write } x = f(t) \text{ and } y = g(t).)$$

This pair of functions is called the **parametric equations** of the curve.

And the variable t is called a **parameter**.

Note: The domain of t is often $(-\infty, \infty)$ or an interval $a \leq t \leq b$.

Goal 1: *Understand* parametric curves. (Today)

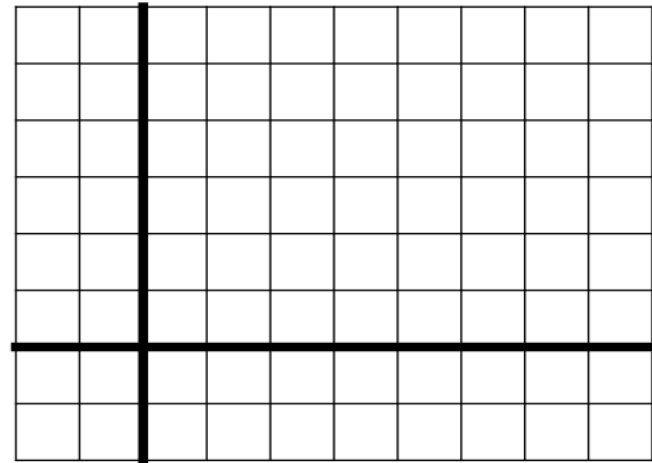**Goal 2:** *Do calculus* using parametric curves. (Next time)

Sketching Parametric Curves

What is the shape of a curve given by parametric equations?

▶ By hand   ⟵ How do you plot $y = f(x)$?

▶ Use a calculator or computer

Example. Plot the curve defined
by $x(t) = t^2 - 2t$ and $y(t) = t + 1$.

t	−1	0	1	2	3	4
$x(t)$						
$y(t)$						

The shape of the curve is _____.

Should we have known this?

Key concept: Eliminate the parameter t to combine $x = f(t)$ and
$y = g(t)$ into a "normal" function $y = F(x)$ or $x = F(y)$.

Solve for t in second equation: $t = y - 1$ and plug in:
$x = (y - 1)^2 - 2(y - 1) = y^2 - 4y + 3$, a "sideways parabola".

Around and Around

Example. Plot the curve defined by $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.
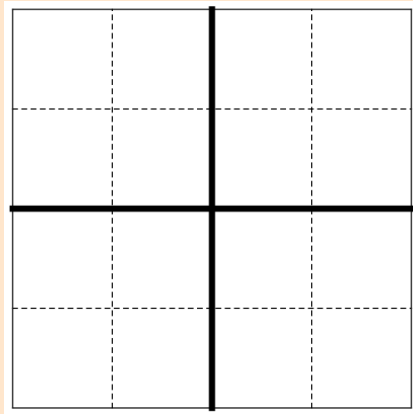
Plot points or solve directly.

$y = \sin(\cos^{-1} x)$
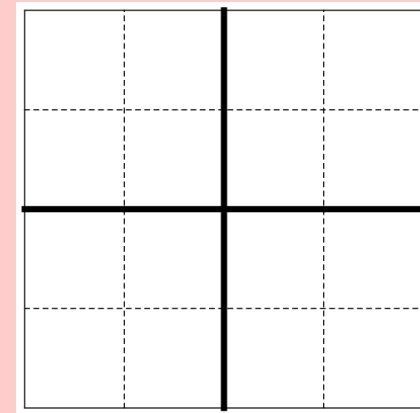$y = \sqrt{1 - x^2}$
$y^2 = 1 - x^2$
$x^2 + y^2 = 1$

A circle! But you knew that.
$\cos^2 t + \sin^2 t = 1$

Starts at $t = 0$: $(1, 0)$ and goes around counterclockwise.

Example. Is this the same as $x = \cos 2t$, $y = \sin 2t$, $0 \le t \le 2\pi$?

Question: What is $x^2 + y^2$?

The figures traced out (the curves) are the same but the functions **are not** the same.

You need to know your trig functions and values at certain points!!!!

Circumnavigation

If we want to draw a circle at some other place

$$(x - h)^2 + (y - k)^2 = r^2,$$

set $\quad x - h = r \cos t \quad$ and $\quad y - k = r \sin t.$

In other words, use the parametric equations

$$x(t) = r \cos t + h \quad \text{and} \quad y(t) = r \sin t + k.$$

Try it out! Get out your graphing calculator TI-(≤ 86).

Switch to Parametric mode: `MODE` $\downarrow \downarrow \downarrow$ `PAR (Enter)`.

Set the domain of T to be from 0 to 2π.
`WINDOW: Tmin = 0, Tmax = 2`π`, Tstep =` π`/10`.

Enter the equations `X`$_1$ `= 3 cos(T) + 2` and `Y`$_1$ `= 3 sin(T) + 4`.

This plots a circle of radius 3 centered at $(2, 4)$.

Computers to the rescue

Calculators and computers can graph much more complicated curves.

$$x_1(t) = t + 2\sin(2t) \quad \text{and} \quad y_1(t) = t + 2\cos(5t)$$

$$x_2(t) = 1.5\cos t - \cos 30t \quad \text{and} \quad y_2(t) = 1.5\sin t - \sin 30t$$

$$x_3(t) = \sin(t + \cos 100t) \quad \text{and} \quad y_3(t) = \cos(t + \sin 100t)$$

Tools:

▶ Wolfram Alpha `http://www.wolframalpha.com/`

▶ More powerful is Wolfram *Mathematica*. Get license from MyQC:
`myqc.qc.cuny.edu/Academics/mathematics/Pages3/access.aspx`

▶ Online plotter: `desmos.com`   Put $(f(t), g(t))$ in parentheses.
`https://www.desmos.com/calculator/ndgy5rppqh`

Next time: What is the shape of a parametric curve? What is the length of a parametric curve? What about polar coordinates?

Before then: Work on homework to present in class Wednesday. Email me contact info, do syllabus quiz. Play with parametric eqns.