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Important: $\frac{d^2y}{dx^2} \neq \frac{d^2y}{dt} \Big/ \frac{d^2x}{dt}$!!!!!

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Question: So what is the tangent line there?

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Put it all together:

t	x	y
-3	9	-3
-1	1	2
0	0	0
1	1	-2
3	9	3

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Also: `desmos.com` or Mathematica

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Example. Find the slope of the tangent line to the curve $r = 2\sin\theta$ at cartesian coordinates $(x, y) = (2, 0)$.