

1. (15 pts) Suppose we are trying to find the global extrema of the function

$$f(x, y, z) = x^2 + xy + y + z^3 - 3z$$

on the ball $x^2 + y^2 + z^2 \leq 9$.

- (a) Find all critical points of f on the **interior** of the ball.
(b) Set up **but do NOT evaluate** the system of equations that would need to be solved in order to apply the method of Lagrange multipliers to find extrema on the **boundary** of the ball.

2. (5 pts) Draw the domain over which $\int_{\pi/4}^{\pi} \int_1^3 r \, dr \, d\theta$ is integrated.

3. (10 pts) Evaluate this integral **after reversing** the order of integration.

$$\int_0^9 \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{x^2} \, dx \, dy$$

4. (15 pts) Consider the spherical cone defined by $\sqrt{x^2 + y^2} \leq z \leq \sqrt{4 - x^2 - y^2}$.

- (a) Set up **but do NOT evaluate** an integral in **cylindrical** coordinates that calculates the **volume** of this region.
(b) Draw in your blue book the projection of this region onto the yz -plane, labeling the boundaries of this region by the equations of the corresponding curves.

5. (15 pts) Find the average value of the function $h(x, y, z) = x$ over the eighth of a sphere of radius 4 in the orthant where all of x , y and z are negative.