Coordinates, Now with More Dimensions....

2D Coordinates

Variables: x (indep), y (dep)
Axes: x-axis $\perp$ y-axis

3D Coordinates

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Coordinates of a point: (a, b)

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Continued Comparison

2D Equations

Lines:

$x = 2$ defines

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Think Pair Share: What region in $\mathbb{R}^3$ do these inequalities give?

$$1 \leq x^2 + y^2 + z^2 \leq 9 \qquad \& \qquad x \geq 0$$

Vectors

Definition: A **vector** is a quantity that has both magnitude and direction, often represented by an arrow. We'll use either $\mathbf{v}$, $\vec{v}$, or $\vec{\mathbf{v}}$.

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- Can have any number of dimensions.

What can we do with vectors?

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Grasping the magnitude of the situation

To write a vector in coordinates, place the tail at the origin and find the coordinates of the head.

- $\vec{\mathbf{u}} = \langle 1, 4, 3 \rangle$,
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- $\vec{\mathbf{u}} = \langle 1, 4, 3 \rangle$, then $|\vec{\mathbf{u}}| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26}$.
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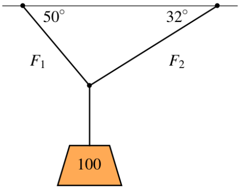
- Example. Unit vector of $\langle 2, -1, -2 \rangle$ is

Vectors are the best way to understand Physics

Example. A 100 lb weight hangs from the ceiling. How much force is held by each rope?



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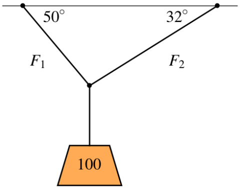

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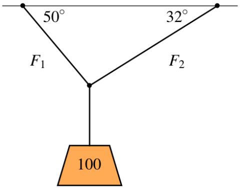

Example. A 100 lb weight hangs from the ceiling. How much force is held by each rope?

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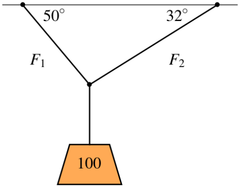

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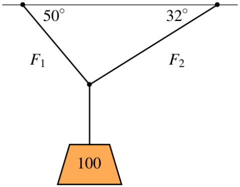

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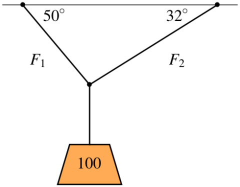

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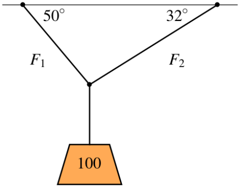

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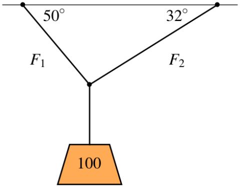

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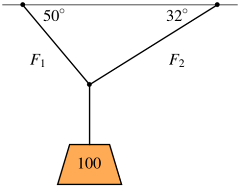

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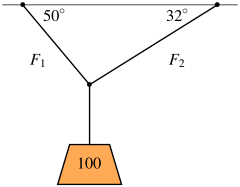

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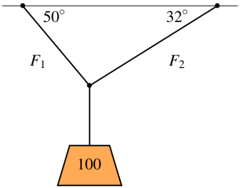

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$$-\cos 50\, F_1 + \cos 32\, F_2 = 0$$

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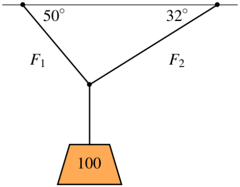

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  - The first force $\vec{\mathbf{F}}_1 = F_1 \langle \underline{\hspace{4cm}} \rangle$ (magnitude · direction)
  - The second force $\vec{\mathbf{F}}_2 = F_2 \langle \underline{\hspace{4cm}} \rangle$
- Use equilibrium to get a system of equations, solve.

$-\cos 50\, F_1 + \cos 32\, F_2 = 0$ and $\sin 50\, F_1 + \sin 32\, F_2 - 100 = 0$

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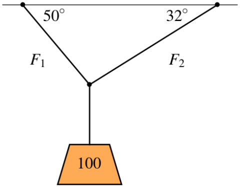

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- Use equilibrium to get a system of equations, solve.

$-\cos 50\, F_1 + \cos 32\, F_2 = 0$ and $\sin 50\, F_1 + \sin 32\, F_2 - 100 = 0$

Solving gives $F_1 \approx 85$ lb and $F_2 \approx 65$ lb.