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Two Lines

In three dimensions, two lines can

- be parallel
- intersect
- be skew

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Answer: In other words, ______________________________.

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$|\vec{P}_1 - \vec{P}_0| = \frac{17}{38}\sqrt{2^2 + 3^2 + (-5)^2} = \frac{17}{\sqrt{38}}$.