

Drawing simple 3-D surfaces

Definition: **Cylinders** are surfaces where all slices are the same.

Example. $z = x^2$. \longleftarrow y is NOT in this equation; y can be anything. For any choice of $y = k$ (parallel to ____-plane), the surface looks like a parabola opening toward the positive z -axis. It is a parabolic cylinder.

Example. $y^2 + z^2 = 1$. \longleftarrow x is not in this equation. For any choice of $x = k$, the surface looks like a unit circle.

Quadric surfaces

Definition: A **quadric surface** is defined by an equation of the form:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0.$$

They are the analog of conic sections in two dimensions.

Through rotation or translation, we need only consider two types:

$$Ax^2 + By^2 + Cz^2 + J = 0 \quad \text{and} \quad Ax^2 + By^2 + Iz = 0.$$

Strategy: Take slices in each coordinate direction, piece the slices together to understand the surface.

$$\begin{cases} x = \text{constant } k \\ y = \text{constant } k \\ z = \text{constant } k \end{cases}$$

Example. $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1.$

When $z = 0$, $x^2 + \frac{y^2}{9} = 1$ is an ellipse. ($-2 \leq k \leq 2$)

When $z = k$, $x^2 + \frac{y^2}{9} = 1 - \frac{k^2}{4}$ is an ellipse when $1 - \frac{k^2}{4} \geq 0$.

When $x = k$, $\frac{y^2}{9} + \frac{z^2}{4} = 1 - k^2$ is an ellipse

When $y = k$, $x^2 + \frac{z^2}{4} = 1 - \frac{k^2}{9}$ is an ellipse

Every slice is an ellipse \rightsquigarrow surface is an ellipsoid.

Example. $z = y^2 - x^2$

Slices

$x = k$

$y = k$

$z = k$

Eqn Format

$z = y^2 - k^2$

$z = k^2 - x^2$

$k = y^2 - x^2$

Conic section

Sketches

Assemble together:

Need to know

- ▶ There are six different families of quadric surfaces.

Ellipsoid (Sphere)

$$+ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Cone

$$+ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

Elliptic paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Hyperboloid of one sheet

$$+ \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Hyperbolic paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Hyperboloid of two sheets

$$- \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

- ▶ Matching equations to surfaces.
- ▶ More variety than conic sections but same building blocks.
- ▶ How to find slices, assemble to a rough sketch.

Online Resources:

<https://www.youtube.com/watch?v=LBii0EiD3Yk>

<http://tutorial.math.lamar.edu/Classes/CalcIII/QuadricSurfaces.aspx>