

# Functions of Several Variables

## Function of one variable

$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$f : x \mapsto f(x)$$

$f$  takes in a real number  $x$   
 outputs real number  $y = f(x)$   
**Domain:**  $x$ -vals where  $f$  defined.  
**Range:**  $y$ -vals that  $f$  can output.

## Function of several variables

$$f : \mathbb{R}^2 \rightarrow \mathbb{R} \quad (\text{or } \mathbb{R}^n \rightarrow \mathbb{R})$$

$$f : (x, y) \mapsto f(x, y) = z$$

$f$  takes in two real numbers  $x$  &  $y$   
 outputs real number  $z = f(x, y)$   
**Domain:**  $(x, y)$ -vals where  $f$  defined.  
**Range:**  $z$ -vals that  $f$  can output.

Three ways to understand functions of two variables:

- ▶ What is the domain of the function? (A set in 2D)
- ▶ Sketching the graph of a function. (A surface over this set)
- ▶ Drawing the level curves of the function (A set of 2D curves)
  - ▶ A curve represents points in the domain at the same “height” .

## The domain of a function of two variables

**Example.** What is the domain of the functions

$$f(x, y) = \frac{\sqrt{x + y + 1}}{x - 1} \quad \text{and} \quad g(x, y) = x \ln(y^2 - x)?$$

**Example.** Sketch the following functions

$$f(x, y) = 6 - 3x - 2y \quad \text{and} \quad g(x, y) = \sqrt{9 - x^2 - y^2}.$$

## Level curves

The **level curves** or **contour curves** of a function  $f$  are the set of curves of the equations  $f(x, y) = k$  for varying constants  $k$ .

1. Temperature maps (isothermals)

Image Search

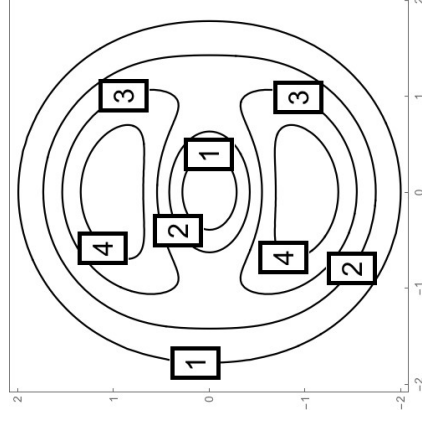
2. Mountain range maps (contour line)

Image Search

- ▶ Visualize level curves being lifted to piece together the surface.
- ▶ Where the lines are close together, the surface is steeper.

**Example.** What surface corresponds to this contour map?

**Example.** Sketch the level curves of the function  $h(x, y) = \sqrt{9 - x^2 - y^2}$  for  $k = 0, 1, 2, 3$ .



## More variables

We can define functions of more variables  $f(x_1, x_2, \dots, x_n)$ .

- ▶ Input:  $n$  numbers. Output: one number
- ▶ The simplest type of function is a linear function:  $f(\vec{x}) = \vec{c} \cdot \vec{x}$ .

$$f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

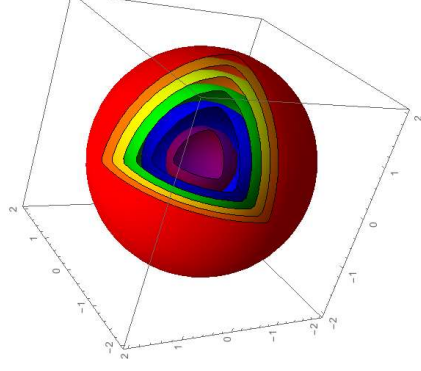
Think:  $c_i$  is the unit cost of object  $i$ . Cost of  $x_i$  units is  $\underline{\hspace{1cm}}$ .

So  $f(\vec{x})$  is the total cost of all objects. (How much was lunch?)

- ▶ Can not visualize the surface when  $n \geq 3$ .
- ▶ When  $n = 3$ , we can understand the **level surfaces** of  $f$ .  
(Where is  $f(x, y, z) = k$ ?)

This gives a **surface** on which the function has a constant value.

**Example.**  $f(x, y, z) = x^2 + y^2 + z^2$



**Think:** Which positions in this room have the same temperature?