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A point a where this is true is called a **critical point**.

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If $f(x, y)$ has a local max or local min at $(x, y) = (a, b)$, **then:**

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Important vocabulary:

- A maximum or minimum: means ________________.
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4. If $D = 0$, the test is inconclusive.

Extreme Examples

Example. Find the local extrema and saddle points of

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- Critical points:

- For each: Find $D(a, b)$, classify.

Absolute (global) Extrema

Functions of one variable

Extreme Value Theorem:
If f is continuous on a **closed interval**, then f attains an absolute max and absolute min **somewhere** on this interval.

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- Parametrize the boundary. (Including endpoints!)

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- Find critical points of 1-Var fcn.
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Largest, smallest function values determine absolute extrema on set.

Optimization is just finding maxima and minima

Example. Find the global extrema of $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $0 \leq x \leq 3$ and $0 \leq y \leq 2$.

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$$\left.\begin{array}{r} 0 = f_x = 2x - 2y \\ 0 = f_y = -2x + 2 \end{array}\right\}$$

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- Find extreme values along boundary. (Piecewise-defined!)

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$f(3, y) = 9 - 4y$ ⇝ min: 1 @ (3,2) max: 9 @ (3,0)

$f(x, 2) = x^2 - 4y + 4$ ⇝ min:__________ max:__________

$f(0, y) = 2y$ ⇝ min: 0 @ (0,0) max: 4 @ (0,2)

- Determine largest & smallest.

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Example. A rectangular box with no lid is made from $12\,\mathrm{m}^2$ of cardboard. What is the maximum volume of the box?

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Example. A rectangular box with no lid is made from 12 m^2 of cardboard. What is the maximum volume of the box?

Solution. Let length, width, and height be x, y, and z, respectively. Then the question asks us to maximize $V =$ ________,
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Solving these simultaneous equations, $12 - 2xy = x^2 = y^2 \Rightarrow x = \pm y$.

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Because this is real world, _____, so we solve $12 - 3x^2 = 0$: _____.

This problem must have an absolute maximum, which must occur at a critical point. (Why?) Therefore $(x, y, z) = (2, 2, 1)$ is the absolute maximum, and the maximum volume is $xyz = 4$.