## Finding slope on a parametric curve

When y is a function of x, what is the slope of the tangent line? For a parametric curve  $\{x = f(t), y = g(t)\}$ , Think of y as a function of x. Then  $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ , so  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$  if \_\_\_\_\_. • Curve has a horizontal tangent where  $\frac{dy}{dt} = 0$  and  $\frac{dx}{dt} \neq 0$ . ▶ Curve has a vertical tangent where  $\frac{dx}{dt} = 0$  and  $\frac{dy}{dt} \neq 0$ . • Question: What is true when  $\frac{dx}{dt} = 0$  AND  $\frac{dy}{dt} = 0$ ? We can use the chain rule again to find  $\frac{d^2y}{dy^2}$ , but be careful!  $y'' = \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx}\right) = \dots \left(\frac{dy}{dx} \text{ is a function of } \dots\right)$ 

## Slope of tangent line

**Example**. What is the tangent line to the curve  $\begin{cases} x(t) = t^2 \\ y(t) = t^3 - 3t \end{cases}$  at (3,0)? *Question:* What is t there?

*Question:* What is the slope there?

*Question:* So what is the tangent line there?

# Sketching the curve

Let's now sketch the curve.

*Question:* Where are there horizontal and vertical tangents?

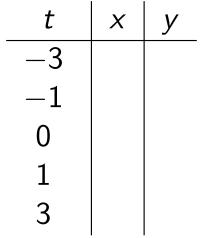
- ► Horizontal:
- Vertical: (Must check?)

*Question:* Where is the curve concave up? concave down?

Calculate  

$$\frac{d^2 y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{3t^2 - 3}{2t}\right)}{\frac{dx}{dt}} = \frac{\frac{(2t)(6t) - (3t^2 - 3)(2)}{4t^2}}{2t} = \frac{\frac{6t^2 + 6}{4t^2}}{2t} = \frac{3(t^2 + 1)}{4t^3}.$$

#### Put it all together:



 $\begin{cases} x(t) = t^2 \\ y(t) = t^3 - 3t \end{cases}$ 

### Polar coordinates

Polar coordinates are an alternate way to think about points in 2D.

Conversions: $x = r \cos \theta$  $r^2 = x^2 + y^2$  $y = r \sin \theta$  $tan \theta = \frac{y}{x}$ 

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin					
cos					
tan					

Need to know Changing coordinates:  $(r, \theta) = (2, -\frac{2\pi}{3})$  then (x, y) =(x, y) = (-1, 1) then  $(r, \theta) =$ Identifying polar equations:  $\theta = 1$ r = 2 $r = 2\cos\theta$  $r = \cos 2\theta$   $r = 1 + \sin \theta$ 

Using your calculator: Switch to Polar mode: MODE  $\downarrow \downarrow \downarrow \downarrow$  POL (Enter). Also:desmos.com or Mathematica

#### Tangents to polar curves

Given a polar curve  $r = f(\theta)$ , we want to know  $\frac{dy}{dx}$ . Just as before, think of y as a function of x. Then  $\frac{dy}{d\theta} = \frac{dy}{dx} \cdot \frac{dx}{d\theta}$ , We conclude:  $\frac{dy}{dx} = \frac{\frac{d}{d\theta}y}{\frac{d}{d\theta}x} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{(r\cos\theta + \sin\theta\frac{dr}{d\theta})}{(r\sin\theta + \cos\theta\frac{dr}{d\theta})}$  if \_\_\_\_\_.

**Example**. Find the slope of the tangent line to the curve  $r = 2 \sin \theta$  at cartesian coordinates (x, y) = (2, 0).