# MATH 245, Spring 2016
# UPDATED Homework 3
# due 10:45am on Monday, April 18.

*Background reading:* Section 1.5, and 5.3A, and these resources:

- About Markov chains:
  http://www.sosmath.com/matrix/markov/markov.html
  http://www.math.bas.bg/~jeni/markov123.pdf

I ask that you do not contact previous Math Modeling students when completing this assignment. Provide details of calculations and assertions that you include. Don't forget to provide acknowledgments for those who helped you with the assignment and those resources that you consulted.

**3-1.** (6 pts) **Determine the reliability of a computer**, where five types of components are required for printing out your group's project: *power*, *desktop*, *input*, *drive*, *printer*. The *power supply* works 99.6% of the time and the *desktop* works 99.9% of the time. There are two types of *input*, one of which must work: a keyboard with reliability 99.99% and a mouse with reliability 99.98%. There are three types of *drives*, one of which must work: the hard drive with reliability 99.7%, the CD-ROM with reliability 99.2%, and the flash drive with reliability 98%. Last, there are two connected *printers*, one of which must work for success: a laser printer with reliability 99.5% and a color printer with reliability 99%.

Last, write a sentence or two discussing the assumptions that are required for your answer and calculations to be correct.

**3-2.** (7 pts) Consider a population of invincible individuals of two types: babies and adults. Consider what happens between time $t$ and time $t+1$. Every adult at time $t$ produces one baby at time $t+1$. Furthermore, every baby at time $t$ grows into an adult at time $t+1$. Adults do not die.

(a) Determine the transition matrix for this situation.

(b) Suppose that at time 0, there is one adult and no babies. Determine the number of adults at time 2 using matrix multiplication by hand.

(c) Now use *Mathematica* to determine the number of adults at time $t$ for $t$ from 0 to 20. What do you notice?

[*Hint: Use Tutorial 5 to learn how to set up the matrix multiplication and the calculation of powers of matrices. I suggest using the* `Table` *command to take care of calculating the sequence of values.*]

**3-3.** (7 pts) Suppose that you are setting up a pizza delivery business with three stores, Alpha Pizza in Flushing, Beta Pizza in Long Island City, and Gamma Pizza in Jamaica.

When a customer calls a store, that store sends out a delivery person, who delivers the pizza and then returns to the closest store. Because of this, the delivery people end up transitioning from store to store throughout the night.

- Suppose that when Alpha Pizza is called, then 1/2 of the time the delivery person returns to Alpha, 1/5 of the time the delivery person goes to Beta, and 3/10 of the time goes to Gamma.
- When Beta Pizza is called, then 3/5 of the time the delivery person returns to Beta and with probability 1/5 the person goes to each of Alpha or Gamma.
- When Gamma Pizza is called, then 3/5 of the time the delivery person returns to Gamma, 4/15 of the time goes to Alpha, and 2/15 of the time goes to Beta.

(a) Set up a Markov Chain model to simulate this situation.

(b) Suppose that one evening there are 5 delivery people at each store at the beginning of the evening and they are all sent out at the same time. What is the expected distribution of the delivery people when they return from this first delivery?

(c) Determine the equilibrium distribution of the delivery people at the end of the day.