Errors inherent in the modeling process

Models always have errors [*mistake, inaccuracy*] $\rightsquigarrow$

- Be aware of them.
- Understand and account for them!
- Include in model discussion.

Frame of reference:

- Formulation: Develop the model.
  - Problem statement is precise and clear. ✓
  - Determine dependent and independent variable(s).
  - Introduce simplifying assumptions.
  - Translate this situation into mathematics.(Have a model.)
- Math. Manipulation: *Mathematical* results from the model.
- Evaluation: A well-thought out discussion.
  - Translate back to the real-world situation, see if it reflects reality.
  - Were the simplifying assumptions good? Based in reality?
  - An honest discussion about the errors inherent to your model.
  - Is the model a good model? (Criteria next class.)

Sidebar: The Mathematical definition of Error

Definition: Suppose you are finding the value of something. Let v be its true value and v_m be the value predicted by a model or measured.

- The **error** is calculated by $v_m - v$.
- The **fractional error** is calculated by $\dfrac{v_m - v}{v}$.
- The **percentage error** is calculated by $\left(\dfrac{v_m - v}{v} \cdot 100\right)\%$.

Example. Suppose that the census measures the 18-22 year old population to be 9,300,000 while the true population is 9,500,000.

- The **error** is
- The **fractional error** is
- The **percentage error** is

Most of the time, we discuss the absolute value of percentage error.

In other words, 5% error means the error is either -5% or 5%.

Formulation Errors (pp. 70–73: Seismology)

1. **Formulation Errors** occur when simplifications or assumptions are made.

Set off an explosion at one place and measure it at another (dist. D). Create a model to determine the depth of a layer in the crust based on the time for the initial explosion to arrive T_1, and the second shock T_2.

$$d = \frac{D}{2}\sqrt{(T_2/T_1)^2 - 1}$$

Assumptions: The earth is flat, and the layer is parallel to the surface.
Error: If layers are not parallel (off by α°), the % errors can be large!

α	1	5	10	30
Model d	1.031	1.031	1.031	1.031
True d	0.997	0.874	0.752	0.504
% error in d	3.4	18	37	105

Important: You can not avoid Formulation Errors.
Scrutinize and discuss all **explicit** and **implicit** assumptions.

Simplifying Assumptions

Problem Statement: Which products should Waldbaums feature at the endcap of Aisle 5 in order to maximize profit?

Simplifying assumptions:

Formulation errors:

Observation Errors

2. **Observation Errors** occur during data collection.

Continuation of the previous example:

Even if the layers are parallel, perhaps our timing is inaccurate.
Let's say that T_1 is 1 second and T_2 is 1.2 seconds,
but that our timer is off by at most 1%.

Then T_1 might be ____ seconds or ____ seconds,
and T_2 might be ____ seconds or ____ seconds.

T_1	over	over	under	under
T_2	over	under	over	under
% error in d	-0.5%	-5%	+6%	0%

Observation Errors in a model can be reduced by measuring many times and taking an average. (Polling averages, Nate Silver)

Computational Errors

3. **Truncation Errors** occur when you approximate an incalculable function.

Question: When is $x^5 + x - 1 = 0$? What is $\sin 1$? Numerically?

Answer: Use a Taylor series approximation:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

4. **Rounding Errors** occur during calculations when your computing device can't keep track of exact numbers.

Question: What is 1.2300001^{10}?

Answer: If we only have three-digit accuracy, then

$1.23 \cdot 1.23 = 1.51$, $1.23 \cdot 1.51 = 1.86$ $\ldots$ $1.23^{10} = \mathbf{7.95}$

$1.2300001 \cdot 1.2300001 = 1.5129002$, $1.2300001 \cdot 1.5129002 = 1.8608674$,

$1.2300001^{10} = \mathbf{7.9259523}$

True answer: $7.92595253991286345258474801873764932003980 5 \cdots$

Descriptively Realistic

Definition: A mathematical model is **descriptively realistic** if the the model is deduced from a correct description of the mechanism involved in what is being modeled.

- Are the simplifying assumptions based off a deep understanding of the underlying situation?

Example. **Full moons.** You observe that in November, December, and January, a full moon appears to occur every 29 days. From these observations, you develop the following model for predicting the full moon in February:

> *The date of the next full moon is 29 days after the date of the last full moon.*

Is this model descriptively realistic? _____ Why?