

Errors inherent in the modeling process

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- ▶ **Evaluation:** A well-thought out discussion.

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- ▶ **Evaluation:** A well-thought out discussion.
 - ▶ Translate back to the real-world situation, see if it reflects reality.
 - ▶ Were the simplifying assumptions good? Based in reality?
 - ▶ An honest discussion about the errors inherent to your model.
 - ▶ Is the model a good model? (Criteria next class.)

Sidebar: The Mathematical definition of Error

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Most of the time, we discuss the **absolute value** of percentage error. In other words, 5% error means the error is either -5% or 5% .

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Set off an explosion at one place and measure it at another (dist. D). Create a model to determine the depth of a layer in the crust based on the time for the initial explosion to arrive T_1 , and the second shock T_2 .

$$d = \frac{D}{2} \sqrt{(T_2/T_1)^2 - 1}$$

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Error: If layers are not parallel (off by α°), the % errors can be large!

α	1	5	10	30
Model d	1.031	1.031	1.031	1.031
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Important: You can not avoid Formulation Errors.

Scrutinize and discuss all **explicit** and **implicit** assumptions.

Simplifying Assumptions

Problem Statement: Which products should Waldbaums feature at the endcap of Aisle 5 in order to maximize profit?

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Observation Errors in a model can be reduced by measuring many times and taking an average. (Polling averages, Nate Silver)

Computational Errors

3. **Truncation Errors** occur when you approximate an incalculable function.

Question: When is $x^5 + x - 1 = 0$? What is $\sin 1$? Numerically?

Answer: Use a Taylor series approximation:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

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Question: What is 1.2300001^{10} ?

Answer: If we only have three-digit accuracy, then

$$1.23 \cdot 1.23 = 1.51, \quad 1.23 \cdot 1.51 = 1.86 \quad \dots \quad 1.23^{10} = \mathbf{7.95}$$

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$$1.2300001 \cdot 1.2300001 = 1.5129002, \quad 1.2300001 \cdot 1.5129002 = 1.8608674,$$

$$1.2300001^{10} = \mathbf{7.9259523}$$

True answer: 7.925952539912863452584748018737649320039805...

Descriptively Realistic

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The date of the next full moon is 29 days after the date of the last full moon.

Is this model descriptively realistic? _____ Why?