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Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

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Once you have written the computer program,

Verify that the simulation models the current real-world situation

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- How do the data change?
- Is the alternate scheme better or worse?
- Determine how to implement to cause minimal disruption.

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- You have to build it. (Expensive to develop!)
- Requires computing power and time.
- Makes you over-confident in the results.
- Dealing with probability, so results will always be of the form: "With 95% probability, the wait time will be less than 2 minutes."

Simulating flipping a coin

Goal: Use a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use these *Mathematica* commands:

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The numbers produced by a random number generator are never truly random because they are produced by an algorithm on a deterministic machine.

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In[1]: CoinFlips = RandomInteger[{0,1},20]
Out[1]: {1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1}
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Running the commands again will simulate another trial of 20 flips.

If statements and For loops

In order to incorporate more complex aspects into the model, use If statements and For loops.

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If[condition,t,f]
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Examples of conditions:

`x < 0`   `(x == 0) && (y != 1)`   `RandomInteger[] == 1`

Note the **double equals** sign `==` and **not equals** `!=`.

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- `If[x < 0, -x, x]` is the ________________ function.
- `If[RandomInteger[] == 1, "Head", "Tail"]`:

Using `If` statements in `Table` commands

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To model this is *Mathematica*, use an If statement.

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Alternatively, do this is one step:

```
If[RandomReal[] <= 0.075, 1, 0]
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Using If statements in Table commands

That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

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trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
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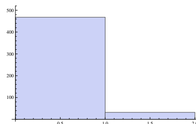
- Alternatively, `Tally[trials]` gives how many times distinct entries appear. Output: `{{0, 468}, {1, 32}}`
- Last, we might want a visualization; Use `Histogram[trials]` to get:



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- Now `i` $= 1$, which is still < 4. So '`Print[i]`' is evaluated and `i` is incremented. Similarly for `i` $= 2$ and `i` $= 3$. Now `i` is incremented to 4, which is NOT < 4, and the loop terminates.

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This variable `i` is called a **counter**.
Be careful to name counters wisely! They are defined as variables.

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- Notice the == and also the ; that separates the commands.
- `loopCount` is ONLY a counter; it does not change each step's evaluation.

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- Run the loop 20 times by having loopCount vary from 1 to 20.
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1', output 'Head' **AND** increase 'headCount',
  - If '0', output 'Tail' **AND** increase 'tailCount'.
- After 20 iterations, display 'headCount' and 'tailCount'.

```
headCount=0; tailCount=0;
For[loopCount = 1, loopCount <= 20, loopCount++,
  If[RandomInteger[]==1,
     Print["Head"]; headCount++,          ← Notice the ';'
     Print["Tail"]; tailCount++]]         ← Notice the '++'
{headCount, tailCount}
```

Simulating rolling a biased die

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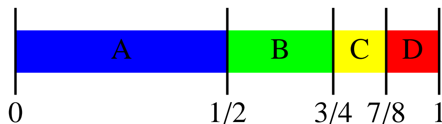

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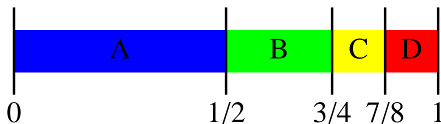
Suppose you have a four-sided die, where the four sides (A, B, C, and D) come up with probabilities 1/2, 1/4, 1/8, and 1/8, respectively.



- Reset the counters: 'aCount=bCount=cCount=dCount=0'.
- For loopCount from 1 to 20,
  - Generate a random real number between 0 and 1.
  - If between 0 and 1/2, then output 'A' and aCount++
    if between 1/2 and 3/4, then output 'B' and bCount++
    if between 3/4 and 7/8, then output 'C' and cCount++
    if between 7/8 and 1, then output 'D' and dCount++
- Display 'aCount', 'bCount', 'cCount', and 'dCount'.

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[loopCount = 1, loopCount <= 20, loopCount++,
 roll=RandomReal[];
 If[ 0  <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1 , Print["d"]; dCount++];]
distribution = {aCount, bCount, cCount, dCount}
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```

- Sample output: (each on its own line)

  a, a, a, d, d, b, a, a, d, a, a, a, a, d, b, a, a, c, a, b $\{12, 3, 1, 4\}$

Simulating rolling a biased die

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- These `If` statements all have no "False" part. (; vs ,)

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- These `If` statements all have no "False" part. (; vs ,)
- Important: You MUST set a variable for the roll. Otherwise, calling `RandomInteger` four times will have you comparing different random numbers in each `If` statement.

Simulating rolling a biased die

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- These `If` statements all have no "False" part. (; vs ,)
- Important: You MUST set a variable for the roll. Otherwise, calling `RandomInteger` four times will have you comparing different random numbers in each `If` statement.
- If you are feeling fancy, you can use one `Which` command instead of four `If` commands.