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Question: Is a game of Candy Land deterministic or probabilistic?

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Example. The roll of the die ... [is '5'] or [is odd] or [is prime] ...

Example. $p(E_1) =$ ______, $p(E_2) =$ ______, $p(E_3) =$ ______.

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- **Subjective guess method:**
  If neither method above applies, give it your best guess.

  Example. How likely is it that your friend will come to a party?

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Example. You wake up and don't know what day it is.

- **Event 1:** Today is a weekday. E_1 vs. E_2
- **Event 2:** Today is cloudy. E_2 vs. E_3
- **Event 3:** Today is Modeling day. E_1 vs. E_3

Independent Events

- When events E_1 (in X_1) and E_2 (in X_2) are *independent* events,

$$p(E_1 \text{ AND } E_2) = p(E_1)p(E_2).$$

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Example. What is the probability that you roll a blue 1 OR a red 6?

This does not work with *dependent* events.

Decision Trees

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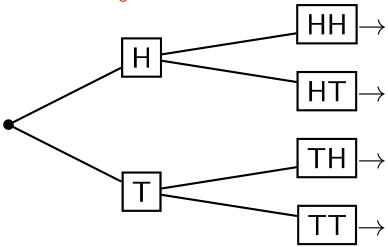

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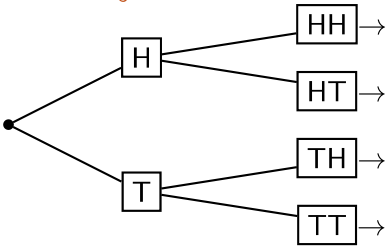

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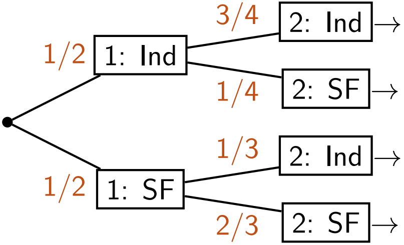
Example. Indiana and SF State U. play two soccer games. (p. 382)



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Expected value / mean

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Definition: The **expected value** or **mean** of a random variable is the sum of the numbers weighted by their probabilities. Mathematically,

$$\mu = \mathbb{E}[X] = p(x_1)r(x_1) + p(x_2)r(x_2) + \cdots + p(x_n)r(x_n).$$

Idea: With probability $p(x_1)$, there is a contribution of $r(x_1)$, etc.

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Question: What is the **interpretation** of this number $\mathbb{E}[X]$?

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When two random variables are on two **independent** experiments, the expected value operation behaves nicely:

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Component Reliability

Many systems consist of components pieced together.

Definition: The **reliability** of a system is its probability of success.

To calculate system reliability, first determine how reliable each component is; then apply rules from probability.

Example. Launch the space shuttle into space with a three-stage rocket.

$$\boxed{\text{Stage 1}} \rightarrow \boxed{\text{Stage 2}} \rightarrow \boxed{\text{Stage 3}}$$

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Let $R_1 = 90\%$, $R_2 = 95\%$, $R_3 = 96\%$ be the reliabilities of Stages 1–3.

p(system success) =
p(S1 success AND S2 success AND S3 success)

Component Reliability

Example. Communicating with the space shuttle.

There are two independent methods in which earth can communicate with the space shuttle

- A microwave radio with reliability $R_1 = 0.95$
- An FM radio, with reliability $R_2 = 0.96$.

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p(system success) $=$ p(MW radio success OR FM radio success)