

Course Notes

Mathematical Models, Fall 2018

Queens College, Math 245

Prof. Christopher Hanusa

<http://qcpages.qc.cuny.edu/~chanusa/courses/245/18/>

What is mathematical modeling?

Is it....

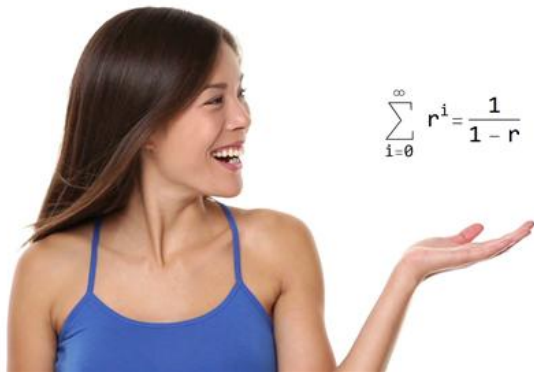
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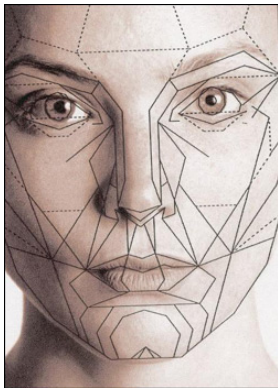
Is it....



No, that's modeling mathematics.

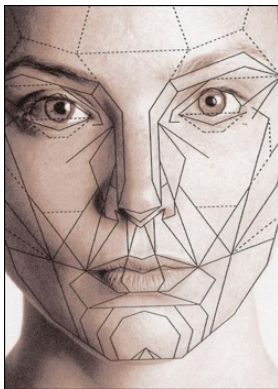
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No, that's the mathematics of modeling.

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No, that's **modeling** mathematical jewelry.

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No, that's a **model airplane**.

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No, that's a **model airplane**.
But we're getting closer.

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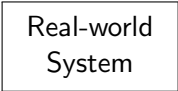
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Then collect data and **analyze our models!**

How do we model?



Real-world
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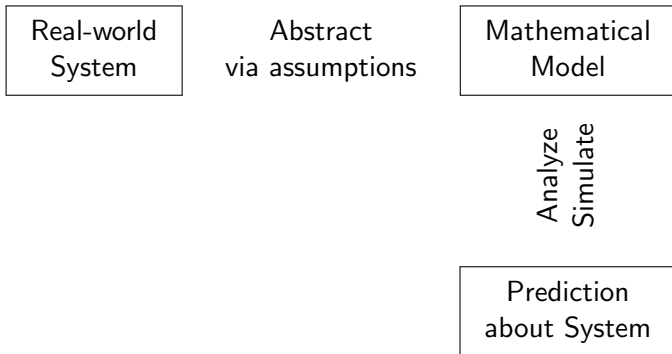
Real-world
System

Abstract
via assumptions

Mathematical
Model

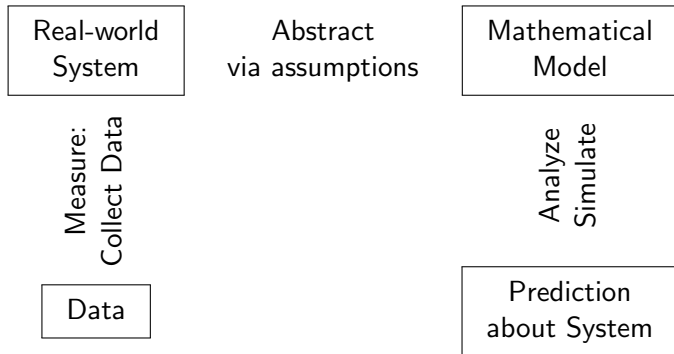
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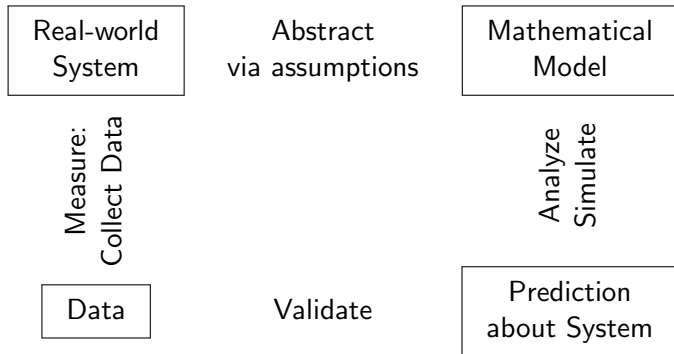
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- ▶ Analyze the model / Create a computer simulation
- ▶ Collect some data from the real world system

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- ▶ Identify the most important variables in a real-world system
- ▶ Analyze the model / Create a computer simulation
- ▶ Collect some data from the real world system
- ▶ Validate your model and refine / revise!

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- ▶ Learning after class
 - ▶ Finish tutorial, review notes, project work

A normal day in this class

Outside
class

- ▶ Preparing for class
 - ▶ Work on homework, do pre-reading, prepare questions.

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- ▶ Arrive on time & Be ready to participate!
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Everything posted online; first one (many parts) due Wednesday.

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- ▶ Label pages with the date and label each question.

Question 1-1. (problem statement here)

Answer the question in complete sentences.

(Leave some space for notes from discussion.)

Question 1-2. (problem statement here)

⋮

Meet the modelers

Group Activity. Get into groups of 3–4 people, with people you don't know. Take some time to get to know them:

- ▶ Introduce yourself. (your name, where you're from, your major)
- ▶ Fill out **the blank side of** your notecard:
 - ▶ Write your name. (Stylize if you wish.)
 - ▶ Write a few words about your name to help me remember.
 - ▶ *Draw* something in the remaining space.
- ▶ Discuss with your groupmates why you wrote what you wrote.
- ▶ Exchange contact information. (phone / email / other)
- ▶ *Small talk suggestion:* What kept you busy this summer?

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Thought Question. There are mathematical models used everyday in the real world. Brainstorm as many as you can. How do they impact your life?

Programming Computers.

We are going to learn modeling and python together.

- ▶ Go to our course webpage:
`qcpages.qc.cuny.edu/~chanusa/courses/245/18/`
- ▶ You'll explore it in detail. > Software.

Key things about Jupyter notebooks:

- ▶ Make a copy of the notebook before any modifications.
- ▶ Each time we start the server, all previous definitions are lost.
- ▶ Jupyter notebooks look linear. They are not.
- ▶ Always evaluate the cells in order from top to bottom.
- ▶ We are using the `modsim` package, so import it each time we open the notebook.
- ▶ `modsim` relies on the `pint` package, so load it first.